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D. Boore's Notes on the Response of Causal and Acausal Butterworth Filters

The response for a causal filter is given by eq. 15.8-6 in Kanasewich (1981):

$$Y = \sqrt{(f/f_c)^{2p}/(1 + (f/f_c)^{2p})}$$

The acausal filter used in the NSMP is obtained by applying the causal filter twice in the time domain (running the filter through the record in opposite directions). The consequence is that the response of the acausal filter is

$$Y = (f/f_c)^{2p}/(1 + (f/f_c)^{2p})$$

The parameter *nroll* was introduced by Converse and Brady (1992), where

$$p = 2 * nroll.$$

Replacing p with *nroll* in the equation above, the low-frequency behavior of the filters goes as

$$Y \rightarrow (f/f_c)^{2*nroll}$$

for the causal filter, whereas the acausal filter falls off more rapidly:

$$Y \rightarrow (f/f_c)^{4*nroll}$$

Also note the difference when $f = f_c$:

$$\text{causal: } Y = 1/\sqrt{2}$$

and

$$\text{acausal: } Y = 1/2$$

This is true no matter what value is chosen for *nroll*. Because of this, it is not possible to make the response of the causal and acausal filters the same. In particular, caution should be used if *nroll* is chosen so as to make the low-frequency asymptotes the same. This requires *nroll* for the acausal filter to be half that of the causal filter. In this case, however, the response for frequencies higher than f_c will be reduced for the acausal as compared to the causal filter (for the same low-frequency behavior, the response of the acausal filter will be systematically lower than the causal filter at higher frequencies).

Choosing *nroll*

It is useful to look at the unfiltered velocity to estimate the filter order needed. A trend given by a polynomial of order n corresponds to a perturbation in acceleration given by a polynomial of order $n-1$, with a Fourier transform going as $1/f^n$ at low frequency (e.g., a linear trend in velocity corresponds to a step in acceleration, with a low-frequency spectrum going as $1/f$). The filter must be chosen such that it decays at low frequencies more quickly than f^n. Note that even the lowest order filter ($nroll = 1$) falls off quickly enough to remove a linear trend in velocity, because the filter goes as

$$Y \rightarrow (f/f_c)^2$$

for the causal filter and

$$Y \rightarrow (f/f_c)^4$$

for the time-domain acausal filter used by the NSMP.

Pros and Cons of Different *nroll*

For larger *nroll*, might be able to use response spectra for a period closer to the cutoff period than for smaller *nroll*. But larger *nroll* will require longer zero pads (for acausal filters) and might produce ringing. I will now compare processing of the Anderson Dam downstream record, component 333, using different *nroll*. This record is used as an example in the BAP manual. Figure 1 shows the acceleration, velocity, and displacement for the two values of *nroll*, for an acausal 0.05 Hz low-cut filter. Note that pre-event pads of 30 sec and 120 sec were used for $nroll = 1$ and $nroll = 4$, respectively. These pad lengths were determined from the formula given in the BAP manual (Converse and Brady, 1992):

$$t_{pad} = 1.5 nroll/f_c.$$

Note that the peak motions are virtually identical for the two values of *nroll*, but that there is pre-event ringing in the displacement record when $nroll = 4$ is used.

Figures 2 and 3 show the displacement response spectra for the two values of *nroll*, as well as the spectrum for the unfiltered record. My rule-of-thumb is that for $nroll = 1$, the response spectrum is relatively unaffected by the filter corner for period less than $0.5T_c$. The equation for acausal filter response predicts that the same filter response is approximately reached for filters 1 and 2 when the following equation relating period and *nroll* is satisfied:

$$(T_1/T_c)^{4nroll_1} = (T_2/T_c)^{4nroll_2}.$$

For the rule-of-thumb, the filter response equals 0.94 for $nroll = 1$ and $T/T_c = 0.5$. The same filter response is obtained when $nroll = 4$ and $T/T_c = 0.84$. Inspecting Figure 3, it seems that the theoretical relation for predicting for which periods the response spectrum is relatively unaffected by the filtering is better for the smaller value of $nroll$. This suggests that advice provided by data-providers for the useable range of response spectral periods may be overly optimistic if those agencies use higher-order filters and base their advice on theoretical filter response.

References

Converse, A. M. and A. G. Brady (1992). BAP — Basic strong-motion accelerogram processing software; Version 1.0, *U.S. Geol. Surv. Open-File Rept. 92-296A*, 174p.

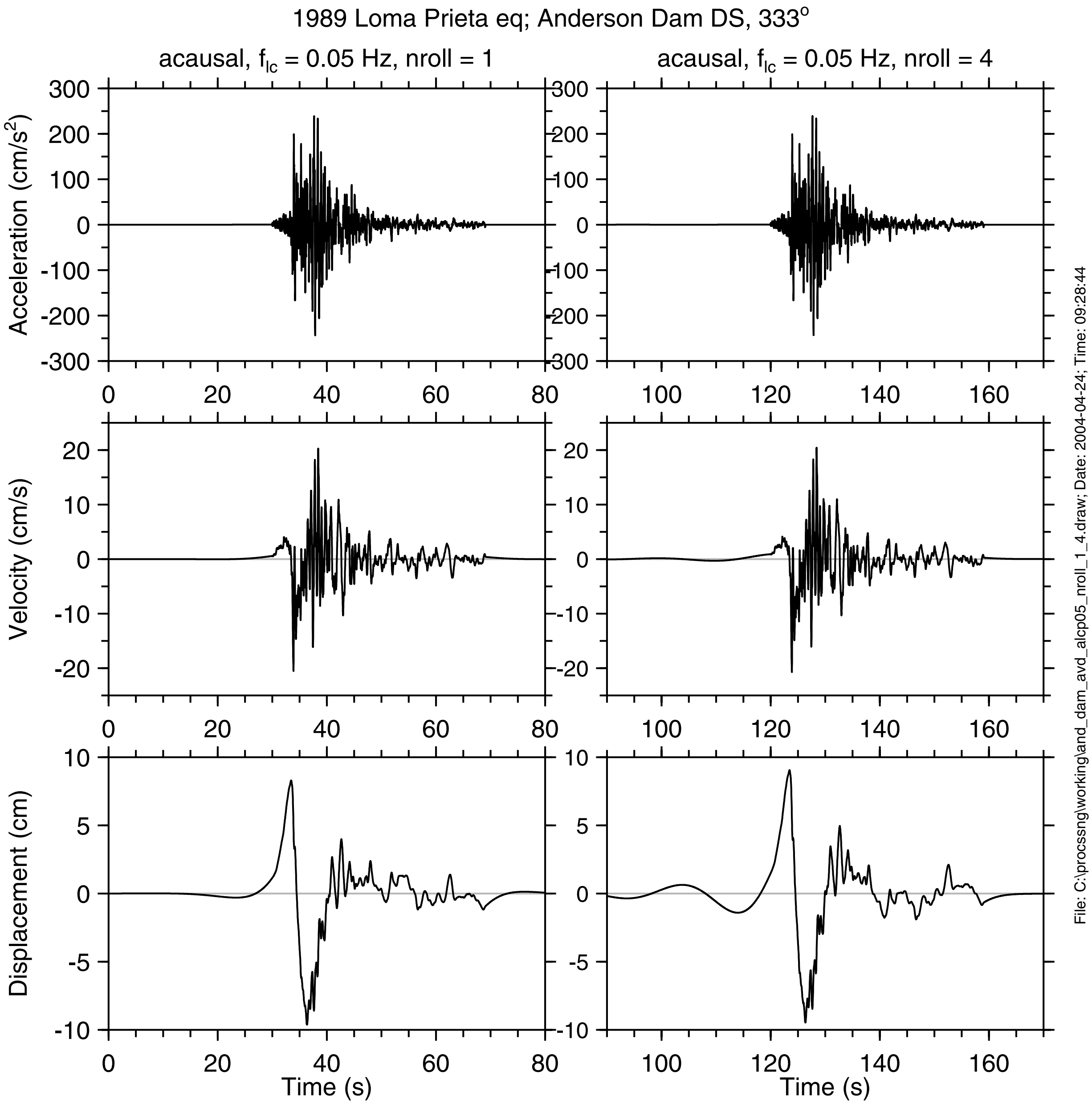

Fig. 1. Comparison of acceleration, velocity, and displacement for an acausal 0.05 Hz low-cut filter with $nroll = 1$ and $nroll = 4$. The time axis includes the pads (30 sec and 120 sec of pre-event pads for the $nroll = 1$ and $nroll = 4$ cases, respectively). Note the ringing in the displacement record for $nroll = 4$.

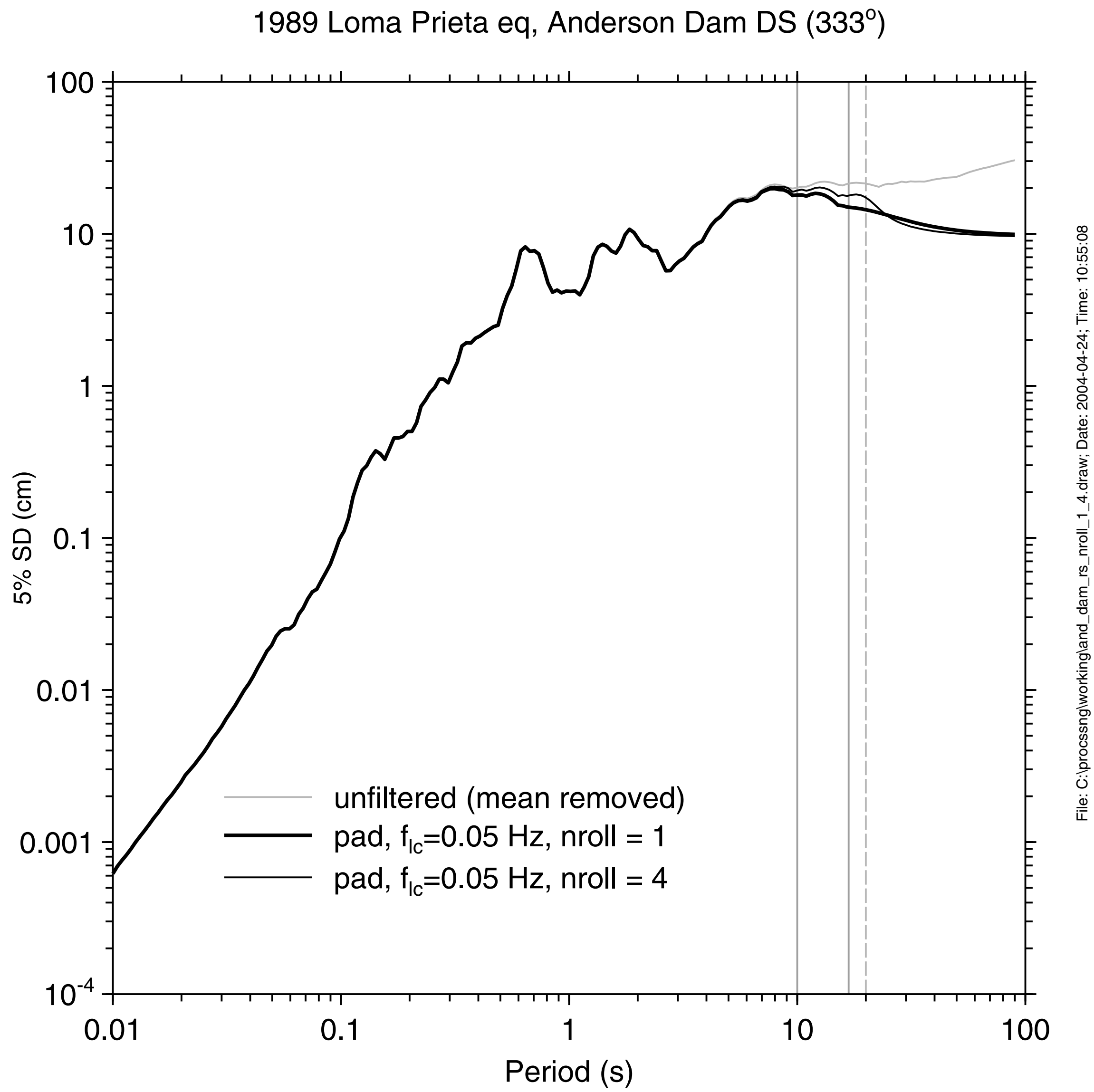

Fig. 2. Comparison of 5%-damped displacement response spectrum for an acausal 0.05 Hz low-cut filter with $nroll = 1$ and $nroll = 4$. Also shown is the displacement response spectrum with no filtering. The left-most solid vertical gray line is drawn for $T/T_c = 0.5$, for which the filter response equals 0.94 when $nroll = 1$; the right-most solid vertical gray line is drawn at a period for which theoretical filter response also equals 0.94 for $nroll = 4$; in this case $T/T_c = 0.84$. The dashed vertical gray line is plotted at the filter corner.

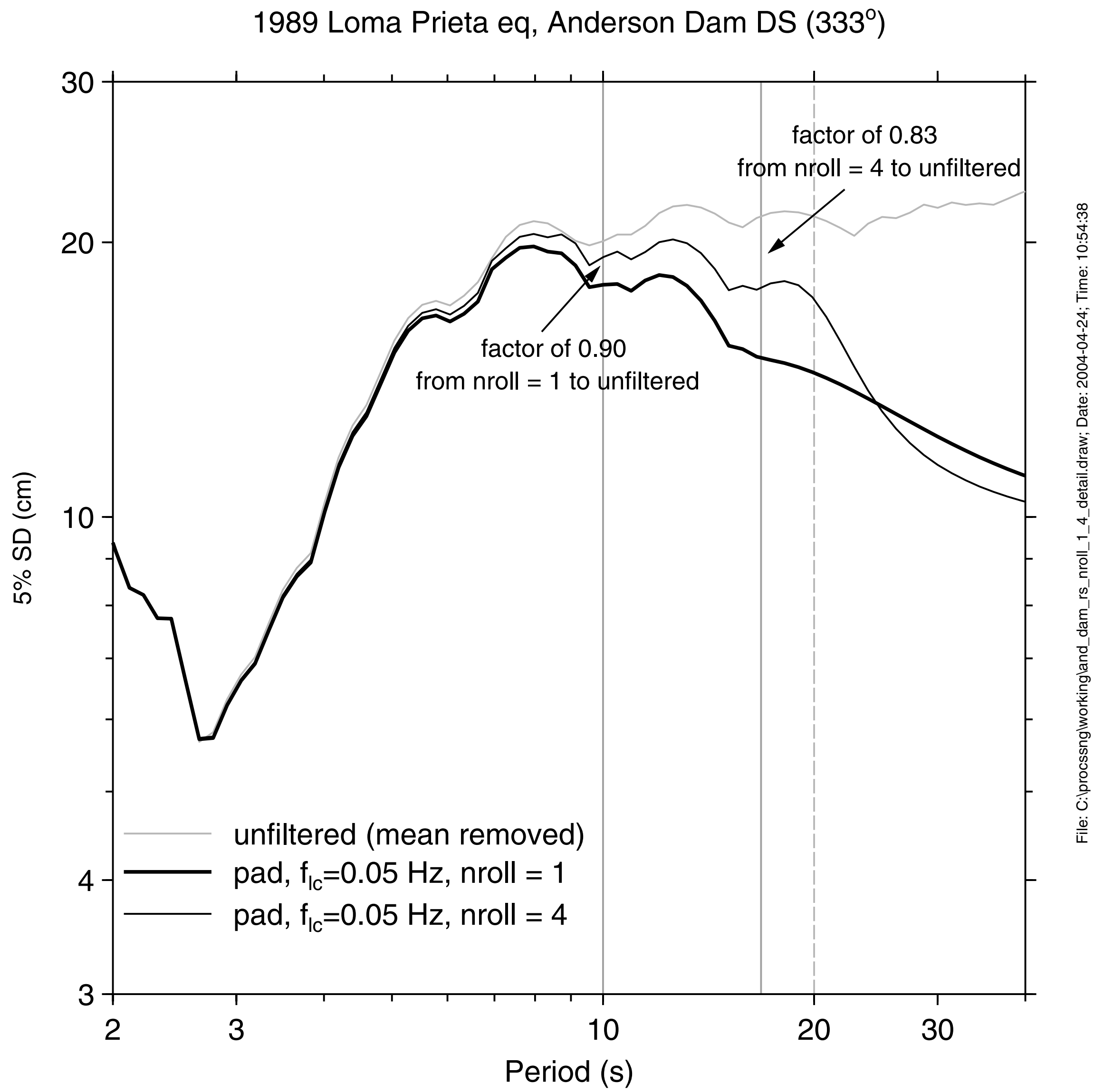

Fig. 3. Comparison of 5%-damped displacement response spectrum for an acausal 0.05 Hz low-cut filter with $nroll = 1$ and $nroll = 4$, showing detail around the filter corner period. Also shown is the displacement response spectrum with no filtering. The left-most solid vertical gray line is drawn for $T/T_c = 0.5$, for which the filter response equals 0.94 when $nroll = 1$; the right-most solid vertical gray line is drawn at a period for which theoretical filter response also equals 0.94 for $nroll = 4$; in this case $T/T_c = 0.84$. The dashed vertical gray line is plotted at the filter corner.