At what period does PSA equal PGA?

Notes by Dave Boore

Date: 30 September 2010

The strict answer to the question posed in the title is when T=0.0 s. This is easy to show from the equation describing the response of a single-degree-of-freedom damped oscillator, where PSA approaches PGA asymptotically as the period approaches 0.0, independent of damping (e.g., see aa_pa_rv_pv_2.pdf in http://www.daveboore.com/daves_notes.php).

The question posed above is not of practical interest. The question should be: At what period is

$$1-\varepsilon < \left|PSA(T)/PGA\right| < 1+\varepsilon \tag{1}$$

for T and all shorter periods (this specification is necessary to avoid the uninteresting situation where PSA crosses the PGA line; this usually happens at periods greater than 0.1 s), where ε is a specified small number. As $PSA > PGA$ seems to be true for all periods (is this always true?), the equation above can be simplified to

$$PSA(T)/PGA < 1+\varepsilon \tag{2}$$

The period for which this equation is satisfied depends on the attenuation of high-frequency energy in the ground motion. As I will show here, except for very hard rock conditions, the common assumption that PSA = PGA when T= 0.01 s is valid (but T=0.02 s for $PSA \approx PGA$ is NOT a good rule of thumb if the approximation is to be within 2%, or perhaps larger (I did not investigate $\varepsilon > 0.02$)). For very hard rock sites (such as those that occur in eastern North America), the period can be as short as 0.002 to 0.004 s. To investigate the question implied by equation (2), I used the SMSIM program gm_rv_drvr (these programs can be obtained from the online software page of www.daveboore.com) to compute PSA for two magnitudes (5.5 and 7.5) and two distances (10 km and 100 km). I used parameters appropriate for hard rock (HR) and soft rock (BC, defined as Vs30 = 760 m/s, which is the boundary between the NEHRP classes B and C) sites in eastern North America (ENA) and for a generic rock sites in western North America (WNA) (Vs30 approximately 620 m/s, using Boore and Joyner (1997)'s generic rock velocity amplifications for WNA). Here are the parameter files used in the computations:

ENA: HR:

```
!Revision of program involving a change in the parameter file on this date:
   12/16/09
!Title:
  Atkinson and Boore (2006) hard-rock parameters
!rho, beta, prtitn, radpat, fs:
    2.8 3.7 0.707 0.55 2.0
```

```
!spectral shape: source number, pf_a, pd_a, pf_b, pd_a
! where source number means:
!  1 = 1-corner (S = 1/(1+(f/fc)**pf_a)**pd_a)
!  2 = Joyner (BSSA 74, 1167--1188)
!  3 = Atkinson (BSSA 83, 1778--1798; see also Atkinson & Boore, BSSA 85,
!      17--30)
!  4 = Atkinson & Silva (BSSA 87, 97--113)
!  5 = Haddon 1996 (approximate spectra in Fig. 10 of
!          Haddon's paper in BSSA 86, 1300--1313;
!          see also Atkinson & Boore, BSSA 78, 917--934)
!  6 = AB98-California (Atkinson & Boore BSSA 78, 917--934)
!  7 = Boatwright & Choy (this is the functional form used by
!                         Boore & Atkinson, BSSA 79, p. 1761)
!  8 = Joyner (his ENA two-corner model, done for the SSHAC elicitation
!      workshop)
!  9 = Atkinson & Silva (BSSA 90, 255--274)
! 10 = Atkinson (2005 model),
! 11 = Generalized two corner model
!      (S = [1/(1+(f/fa)**pf_a)**pd_a]*[1/(1+(f/fb)**pf_b)**pd_b])
! pf_a, pd_a, pf_b, pd_a are used for source numbers 1 and 11, usually
! subject to the constraint pf_a*pd_a + pf_b*pd_b = 2 for an omega-squared
! spectrum. The usual single-corner frequency model uses
! pf_a=2.0,pd_a=1.0; the Butterworth filter shape is given by
! pf_a=4.0,pd_a=0.5.  pf_b and pd_b are only used by source 11, but dummy
! values must be included for all sources.
     1 2.0 1.0 0.0 0.0
!spectral scaling: stressc, dlsdm, fbdfa, amagc, c1_fa, c2_fa, amagc4fa
! (stress=stressc*10.0**(dlsdm*(amag-amagc))
! (fbdfa, amagc for Joyner model, usually 4.0, 7.0)
! c1_fa, c2_fa are the coefficients relating log fa to M in
! source 11, as given by the equation log fa = c1_fa + c2_fa*(M-amagc4fa).
! fb for source 11 is given such that the high-frequency spectral level
! equals that for a single corner frequency model with a stress parameter
! given by stress=stressc*10.0**(dlsdm*(amag-amagc).
! See Tables 2 and 3 in Boore (2003) for various source descriptions
! (Note: the parameters in the line below are not used for most of the
! sources, for which the spectrum is determined by fixed relations between
! corner frequency and seismic moment, but placeholders are still needed)
   140.0 0.0 4.0 6.5  0.0 0.0 0.0
!iflag_h_eff, c1_log10_h_eff, c2_log10_h_eff
!  If iflag = 1, compute an effective depth as
!  h_eff = 10.0**(c1_log10_h_eff + c2_log10_h_eff*amag) and modify the closest
!  distance by this depth: rmod = sqrt(r^2+h_eff^2)); use rmod in
!  the calculations
!  1 -0.05 0.15 ! Atkinson and Silva (2000) values
   0  0.0 0.0
!gsprd: r_ref, nsegs, (rlow(i), a_s, b_s, m_s(i))  (Usually set r_ref = 1.0 km)
    1.0
    3
      1.0 -1.3 0.0 6.5
     70.0 +0.2 0.0 6.5
    140.0 -0.5 0.0 6.5
!q: fr1, Qr1, s1, ft1, ft2, fr2, qr2, s2, c_q
   0.02 1000 0.0 0.2 1.4242 3.02 1272 0.32 3.7
!source duration: weights of 1/fa, 1/fb
    1.0 0.0
!path duration: nknots, (rdur(i), dur(i), slope of last segment)
    4
    0.0 0.0
   10.0 0.0
   70.0 9.6
  130.0 7.8
    0.04
!site amplification: namps, (famp(i), amp(i))
    5
    0.5    1.0
    1.0    1.13
    2.0    1.22
    5.0    1.36
    10.0   1.41
!site diminution parameters: fmax, kappa, dkappadmag, amagkref
```

```
! (NOTE: fmax=0.0 or kappa=0.0 => fmax or kappa are not used.  I included this
!  to prevent the inadvertent use of both fmax and kappa to control the diminution
!  of high-frequency motion (it would be very unusual to use both parameters
!  together.  Also note that if do not want to use kappa, dkappadmag must also
!  be set to 0.0).
    0.0 0.005 0.0 6.0
!low-cut filter parameters: fcut, nslope (=4, 8, 12, etc)
    0.0 4
!rv params: zup, eps_int (int acc), amp_cutoff (for fup), osc_crrctn(1=b&j;2=l&p)
    10.0 0.00001  0.001 1
!window params: idxwnd(0=box,1=exp), tapr(<1), eps_w, eta_w, f_tb2te, f_te_xtnd
    1 0.05 0.2 0.05 2.0 1.0
!timing stuff: dur_fctr, dt, tshift, seed, nsims, iran_type (0=normal;1=uniform)
    1.3  0.002 50.0 123.0 100 0
```

ENA: BC

```
!Revision of program involving a change in the parameter file on this date:
   12/16/09
!Title:
  Atkinson and Boore (2006) soft-rock (BC, Vs30=760 m/s) parameters
!rho, beta, prtitn, radpat, fs:
    2.8 3.7 0.707 0.55 2.0
!spectral shape: source number, pf_a, pd_a, pf_b, pd_a
! where source number means:
!  1 = 1-corner (S = 1/(1+(f/fc)**pf_a)**pd_a)
!  2 = Joyner (BSSA 74, 1167--1188)
!  3 = Atkinson (BSSA 83, 1778--1798; see also Atkinson & Boore, BSSA 85,
!      17--30)
!  4 = Atkinson & Silva (BSSA 87, 97--113)
!  5 = Haddon 1996 (approximate spectra in Fig. 10 of
!          Haddon's paper in BSSA 86, 1300--1313;
!          see also Atkinson & Boore, BSSA 78, 917--934)
!  6 = AB98-California (Atkinson & Boore BSSA 78, 917--934)
!  7 = Boatwright & Choy (this is the functional form used by
!                         Boore & Atkinson, BSSA 79, p. 1761)
!  8 = Joyner (his ENA two-corner model, done for the SSHAC elicitation
!      workshop)
!  9 = Atkinson & Silva (BSSA 90, 255--274)
! 10 = Atkinson (2005 model),
! 11 = Generalized two corner model
!      (S = [1/(1+(f/fa)**pf_a)**pd_a]*[1/(1+(f/fb)**pf_b)**pd_b])
! pf_a, pd_a, pf_b, pd_a are used for source numbers 1 and 11, usually
! subject to the constraint pf_a*pd_a + pf_b*pd_b = 2 for an omega-squared
! spectrum. The usual single-corner frequency model uses
! pf_a=2.0,pd_a=1.0; the Butterworth filter shape is given by
! pf_a=4.0,pd_a=0.5.  pf_b and pd_b are only used by source 11, but dummy
! values must be included for all sources.
     1 2.0 1.0 0.0 0.0
!spectral scaling: stressc, dlsdm, fbdfa, amagc, c1_fa, c2_fa, amagc4fa
! (stress=stressc*10.0**(dlsdm*(amag-amagc))
! (fbdfa, amagc for Joyner model, usually 4.0, 7.0)
! c1_fa, c2_fa are the coefficients relating log fa to M in
! source 11, as given by the equation log fa = c1_fa + c2_fa*(M-amagc4fa).
! fb for source 11 is given such that the high-frequency spectral level
! equals that for a single corner frequency model with a stress parameter
! given by stress=stressc*10.0**(dlsdm*(amag-amagc).
! See Tables 2 and 3 in Boore (2003) for various source descriptions
! (Note: the parameters in the line below are not used for most of the
! sources, for which the spectrum is determined by fixed relations between
! corner frequency and seismic moment, but placeholders are still needed)
   140.0 0.0 4.0 6.5  0.0 0.0 0.0
!iflag_h_eff, c1_log10_h_eff, c2_log10_h_eff
!  If iflag = 1, compute an effective depth as
!  h_eff = 10.0**(c1_log10_h_eff + c2_log10_h_eff*amag) and modify the closest
!  distance by this depth: rmod = sqrt(r^2+h_eff^2)); use rmod in
!  the calculations
!  1 -0.05 0.15 ! Atkinson and Silva (2000) values
   0  0.0 0.0
!gsprd: r_ref, nsegs, (rlow(i), a_s, b_s, m_s(i))  (Usually set r_ref = 1.0 km)
    1.0
```

```
    3
      1.0 -1.3 0.0 6.5
     70.0 +0.2 0.0 6.5
    140.0 -0.5 0.0 6.5
!q: fr1, Qr1, s1, ft1, ft2, fr2, qr2, s2, c_q
   0.02 1000 0.0 0.2 1.4242 3.02 1272 0.32 3.7
!source duration: weights of 1/fa, 1/fb
    1.0 0.0
!path duration: nknots, (rdur(i), dur(i), slope of last segment)
    4
    0.0 0.0
   10.0 0.0
   70.0 9.6
  130.0 7.8
    0.04
!site amplification: namps, (famp(i), amp(i))
    14
    0.0001 1.000
    0.1014 1.073
    0.2402 1.145
    0.4468 1.237
    0.7865 1.394
    1.3840 1.672
    1.9260 1.884
    2.8530 2.079
    4.0260 2.202
    6.3410 2.313
    12.540 2.411
    21.230 2.452
    33.390 2.474
    82.000 2.497
!site diminution parameters: fmax, kappa, dkappadmag, amagkref
! (NOTE: fmax=0.0 or kappa=0.0 => fmax or kappa are not used.  I included this
!  to prevent the inadvertent use of both fmax and kappa to control the diminution
!  of high-frequency motion (it would be very unusual to use both parameters
!  together.  Also note that if do not want to use kappa, dkappadmag must also
!  be set to 0.0).
    0.0 0.02 0.0 6.0
!low-cut filter parameters: fcut, nslope (=4, 8, 12, etc)
    0.0 4
!rv params: zup, eps_int (int acc), amp_cutoff (for fup), osc_crrctn(1=b&j;2=l&p)
    10.0 0.00001  0.001 1
!window params: idxwnd(0=box,1=exp), tapr(<1), eps_w, eta_w, f_tb2te, f_te_xtnd
    1 0.05 0.2 0.05 2.0 1.0
!timing stuff: dur_fctr, dt, tshift, seed, nsims, iran_type (0=normal;1=uniform)
    1.3  0.002 50.0 123.0 100 0
```

WNA: GENERIC ROCK, ATKINSON AND SILVA (2000) MODEL:

```
!Revision of program involving a change in the parameter file on this date:
   12/16/09
!Title
Params for AS2000 (Atkinson and Silva, BSSA 90, 255--274) source, applied to WNA
!rho, beta, prtitn, radpat, fs:
    2.8 3.5 0.707 0.55 2.0
!spectral shape: source number, pf_a, pd_a, pf_b, pd_b
! where source number means:
!  1 = 1-corner (S = 1/(1+(f/fc)**pf_a)**pd_a)
!  2 = Joyner (BSSA 74, 1167--1188)
!  3 = Atkinson (BSSA 83, 1778--1798; see also Atkinson & Boore, BSSA 85,
!      17--30)
!  4 = Atkinson & Silva (BSSA 87, 97--113)
!  5 = Haddon 1996 (approximate spectra in Fig. 10 of
!          Haddon's paper in BSSA 86, 1300--1313;
!          see also Atkinson & Boore, BSSA 88, 917--934)
!  6 = AB98-California (Atkinson & Boore BSSA 88, 917--934)
!  7 = Boatwright & Choy (this is the functional form used by
```

```
!                          Boore & Atkinson, BSSA 79, 1736--1761, p. 1761)
!  8 = Joyner (his ENA two-corner model, done for the SSHAC elicitation
!      workshop)
!  9 = Atkinson & Silva (BSSA 90, 255--274)
! 10 = Atkinson (2005 model),
! 11 = Generalized two corner model
!      (S = [1/(1+(f/fa)**pf_a)**pd_a]*[1/(1+(f/fb)**pf_b)**pd_b])
! pf_a, pd_a, pf_b, pd_a are used for source numbers 1 and 11, usually
! subject to the constraint pf_a*pd_a + pf_b*pd_b = 2 for an omega-squared
! spectrum. The usual single-corner frequency model uses
! pf_a=2.0,pd_a=1.0; the Butterworth filter shape is given by
! pf_a=4.0,pd_a=0.5.  pf_b and pd_b are only used by source 11, but dummy
! values must be included for all sources.
     9 2.0 1.0 0.0 0.0
!spectral scaling: stressc, dlsdm, fbdfa, amagc, c1_fa, c2_fa, amagc4fa
! (stress=stressc*10.0**(dlsdm*(amag-amagc))
! (fbdfa, amagc for Joyner model, usually 4.0, 7.0)
! c1_fa, c2_fa are the coefficients relating log fa to M in
! source 11, as given by the equation log fa = c1_fa + c2_fa*(M-amagc4fa).
! fb for source 11 is given such that the high-frequency spectral level
! equals that for a single corner frequency model with a stress parameter
! given by stress=stressc*10.0**(dlsdm*(amag-amagc).
! See Tables 2 and 3 in Boore (2003) for various source descriptions
! (Note: the parameters in the line below are not used for most of the
! sources, for which the spectrum is determined by fixed relations between
! corner frequency and seismic moment, but placeholders are still needed)
    100.0 0.0 4.0 7.0  0.0 0.0 0.0
!iflag_h_eff, c1_log10_h_eff, c2_log10_h_eff
!  If iflag = 1, compute an effective depth as
!  h_eff = 10.0**(c1_log10_h_eff + c2_log10_h_eff*amag) and modify the closest
!  distance by this depth: rmod = sqrt(r^2+h_eff^2)); use rmod in
!  the calculations
!  1 -0.05 0.15 ! Atkinson and Silva (2000) values
   0  0.0 0.0
!gsprd: r_ref, nsegs, (rlow(i), a_s, b_s, m_s(i))  (Usually set r_ref = 1.0 km)
    1.0
    2
      1.0 -1.0 0.0 6.5
     40.0 -0.5 0.0 6.5
!q: fr1, Qr1, s1, ft1, ft2, fr2, qr2, s2, c_q
    1.0 180 0.45 1.0 1.0 1.0 180 0.45 3.5
!source duration: weights of 1/fa, 1/fb
    0.5 0.0
!path duration: nknots, (rdur(i), dur(i), slope of last segment
    1
      0.0 0.0
     0.05
!site amplification: namps, (famp(i), amp(i))
    11
       0.01                 1.00
       0.09                 1.10
       0.16                 1.18
       0.51                 1.42
       0.84                 1.58
       1.25                 1.74
       2.26                 2.06
       3.17                 2.25
       6.05                 2.58
      16.6                  3.13
      61.2                  4.00
!site diminution parameters: fmax, kappa, dkappadmag, amagkref
! (NOTE: fmax=0.0 or kappa=0.0 => fmax or kappa are not used.  I included this
!  to prevent the inadvertent use of both fmax and kappa to control the diminution
!  of high-frequency motion (it would be very unusual to use both parameters
!  together.  Also note that if do not want to use kappa, dkappadmag must also
! be set to 0.0).
    0.0 0.03 0.0 0.0
!low-cut filter parameters: fcut, nslope (=4, 8, 12, etc)
    0.0 8
!rv params: zup, eps_int (int acc), amp_cutoff (for fup), osc_crrctn(1=b&j;2=l&p)
    10.0 0.00001  0.001 1
```

```
!window params: idxwnd(0=box,1=exp), tapr(<1), eps_w, eta_w, f_tb2te, f_te_xtnd
    1 0.05 0.2 0.05 2.0 2.0
!timing stuff: dur_fctr, dt, tshift, seed, nsims, iran_type (0=normal;1=uniform)
    1.3  0.005 20.0 123.0 10 0
```

Here are plots of the ratios of PSA to PGA, with lines at levels of 1.005, 1.010, and 1.020:

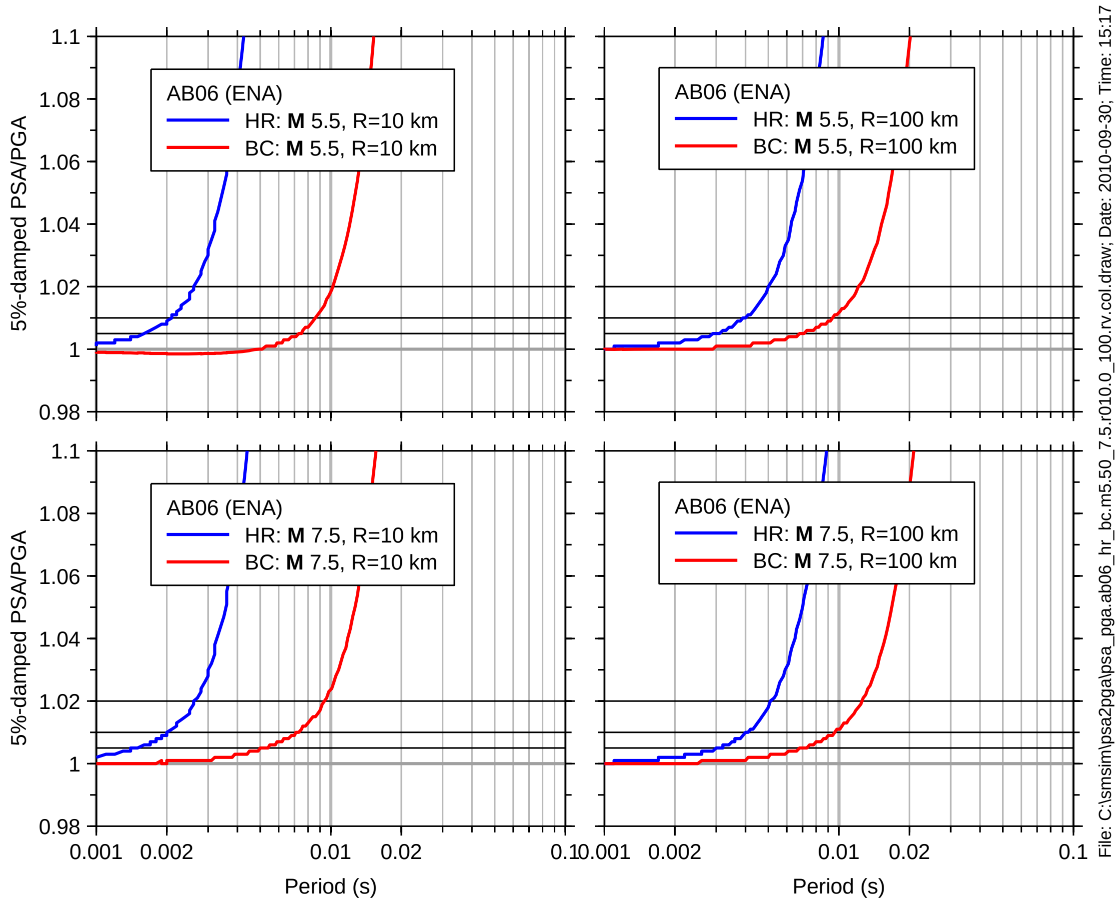

Figure 1. Ratios for ENA

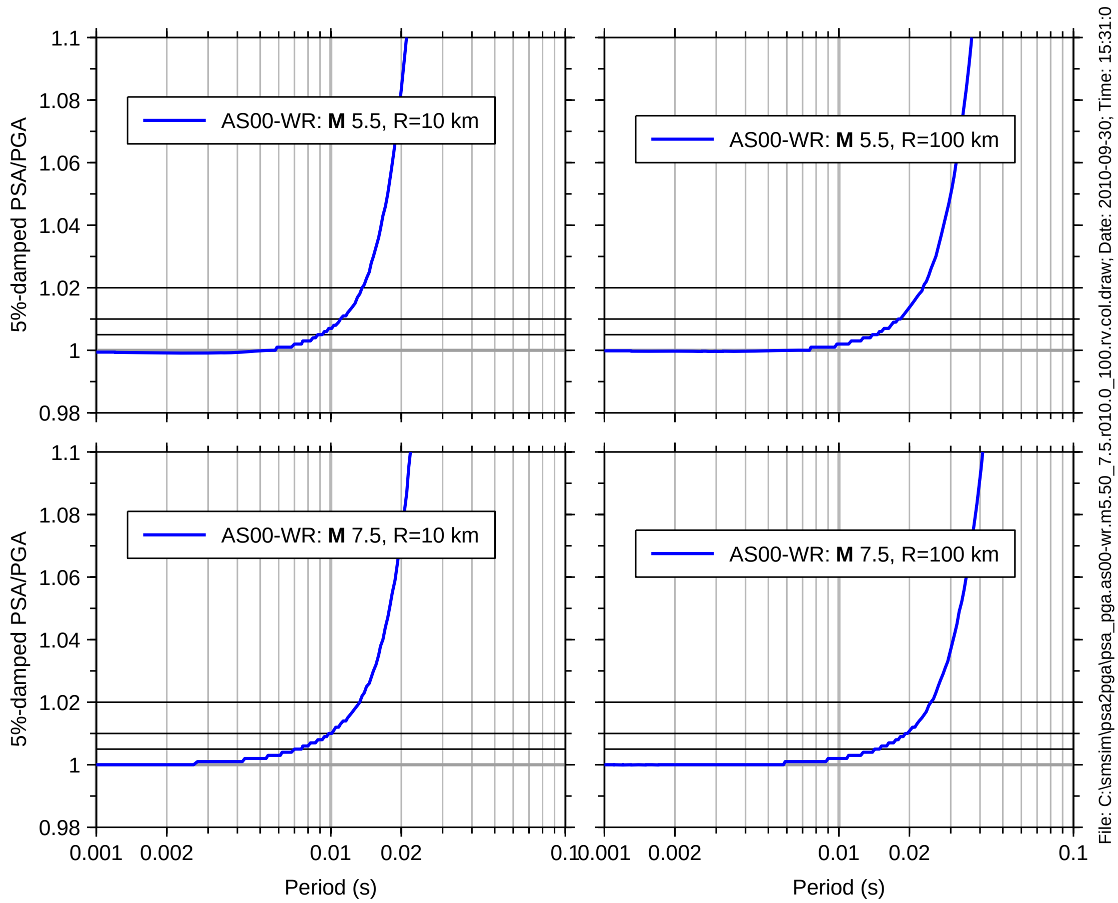

Figure 2. Ratios for WNA

To summarize the results, I picked off the period for which the PSA/PGA is less than 1.005, 1.01, and 1.02. The results for ENA and WNA are in the following tables:

Table 1. Periods (and corresponding frequencies) for which equation (2) is satisfied, for ENA.

M	R	Site	kappa	T(e=0.005)	T(e=0.01)	T(e=0.02)	f(e=0.005)	f(e=0.01)	f(e=0.02)
5.5	10	HR	0.005	0.0016	0.0021	0.0026	631	482	388
7.5	10	HR	0.005	0.0014	0.0020	0.0026	692	497	383
5.5	100	HR	0.005	0.0030	0.0040	0.0050	338	252	200
7.5	100	HR	0.005	0.0031	0.0040	0.0051	326	247	196
5.5	10	BC	0.02	0.0072	0.0087	0.0102	138	115	98
7.5	10	BC	0.02	0.0052	0.0072	0.0094	193	138	106
5.5	100	BC	0.02	0.0069	0.0093	0.0121	145	107	83
7.5	100	BC	0.02	0.0070	0.0095	0.0126	142	105	79

This table shows the importance of kappa on the period for which equation (2) is satisfied. The critical period is not sensitive to magnitude, and it is somewhat sensitive to distance (I probably should do computations and show the results for R=1000 km as well).   For PSA to be within 1% of PGA requires oscillator frequencies greater than 100 Hz in all cases.

Table 2. Periods (and corresponding frequencies) for which equation (2) is satisfied, for WNA.

M	R	Site	kappa	T(e=0.005)	T(e=0.01)	T(e=0.02)	f(e=0.005)	f(e=0.01)	f(e=0.02)
5.5	10	GR	0.03	0.0090	0.0110	0.0135	112	91	74
7.5	10	GR	0.03	0.0072	0.0100	0.0132	138	100	76
5.5	100	GR	0.03	0.0142	0.0181	0.0227	71	55	44
7.5	100	GR	0.03	0.0145	0.0193	0.0245	69	52	41

For the kappa used for the WNA simulations, PSA is within 1% of PGA when T=0.01 s.

References

Atkinson, G. M. and W. Silva (2000). Stochastic modeling of California ground motions, *Bull. Seismol. Soc. Am.* **90**, 255—274.

Boore, D. M. (2005). SMSIM---Fortran Programs for Simulating Ground Motions from Earthquakes: Version 2.3---A Revision of OFR 96-80-A, U.S. Geological Survey Open-File Report, *U. S. Geological Survey Open-File Report* **00-509**, revised 15 August 2005, 55 pp. Available from the online publications link on http://www.daveboore.com .

Boore, D. M. and W. B. Joyner (1997). Site amplifications for generic rock sites, *Bull. Seism. Soc. Am.* **87**, 327–341.