Relations between corner frequency, source radius, and stress drop

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Comparison of Boore (2003) (actually due to Brune, 1970, 1971) and Allmann and Shearer (2009) (using Eshelby and Madariaga) stress drop—source corner frequency relations

From Boore (2003):

where the constant can be related to the stress drop ($\Delta\sigma$). Following BRUNE (1970, 1971), the corner frequency is given by the following equation:

$$f_0 = 4.9 \times 10^6 \beta_s (\Delta\sigma / M_0)^{1/3} \ , \qquad (4)$$

where f_0 is in Hz, β_s (the shear-wave velocity in the vicinity of the source) in km/s, $\Delta\sigma$ in bars, and M_0 in dyne-cm.

Or

$$\Delta\sigma = M_0 \left(\frac{f_0}{4.9 \times 10^6 \beta_s} \right)^3 \qquad (1)$$

Not mixing units (e.g., β in cm/s, M_0 in dyne-cm, $\Delta\sigma$ in dyne/cm^2, f_0 in s^-1), these are the relations:

From Allmann and Shearer (2009):

individual events. Assuming a circular fault, the stress drop $\Delta\sigma$ can be estimated from the corner frequency f_c of the source spectrum and the seismic moment M_0 using the following relations [*Eshelby*, 1957; *Madariaga*, 1976]:

$$\Delta\sigma = \frac{7}{16}\left(\frac{M_0}{r^3}\right), \ \ f_c = 0.32\frac{\beta}{r}, \ \ \rightarrow \ \ \Delta\sigma = M_0\left(\frac{f_c}{0.42\beta}\right)^3, \qquad (3)$$

where r is the source radius and β is the shear wave velocity near the source. We use a constant β of 3.9 km/s and assume the rupture velocity to be 0.9 β. This assumption of a circular fault may not be accurate for all events, especially for

The $f_c - r$ relation is different than that of Brune, and this leads to a different equation relating $\Delta\sigma - f_c$.

$$\Delta\sigma = 13.5 M_0 \left(\frac{f_c}{\beta} \right)^3 \qquad (2)$$

Boore (2003):

$$\Delta\sigma = 8.5 M_0 \left(\frac{f_0}{\beta_s} \right)^3 \qquad (3)$$

Now considering Boore (2003) only:

More precisely, using the equations in Brune (1970, 1971):

$$\Delta\sigma = 8.47 M_0 \left(\frac{f_0}{\beta_s} \right)^3 \qquad (4)$$

This is a 0.3% difference. To get the more precise relation, I should change my basic relation (eq. 4 in Boore, 2003) to

$$f_0 = 4.906 \times 10^6 \beta_s \left(\Delta\sigma / M_0 \right)^{1/3} \qquad (5)$$

In fact, this is what I use in rv_td_subs.for. Here is a code snippet:

```
      if (numsource .eq. 1) then

* Single corner frequency:
        stress = stressc*10.0**(dlsdm*(amag-amagc))
        fa = (4.906e+06) * beta * (stress/am0)**(1.0/3.0)
        fb = fa
```

Equation for source radius:

$$M_0 = \mu \bar{d} \pi r^2 \tag{6}$$

$$\Delta\sigma = \frac{7\pi}{16} \mu \frac{\bar{d}}{r} \tag{7}$$

Combining:

$$r = \left[\left(\frac{7}{16} \right) \frac{M_0}{\Delta\sigma} \right]^{1/3} \tag{8}$$

In terms of source radius and corner frequency, from equations (4) and (8):

$$r = 0.3724 \beta_s / f_0 \tag{9}$$

With units of r, M_0, and $\Delta\sigma$ of km, dyne/cm, and bars this becomes:

$$r = 7.59 \times 10^{-8} \left(M_0 / \Delta\sigma \right)^{1/3} \tag{10}$$

and

$$\Delta\sigma = 4.372 \times 10^{-22} M_0 / r^3 \tag{11}$$

This gives the following table:

M	Ds	r
3	50	0.15
4	50	0.46
5	50	1.46
6	50	4.61
7	50	14.59
8	50	46.12
3	100	0.12
4	100	0.37
5	100	1.16
6	100	3.66
7	100	11.58
8	100	36.61
3	200	0.09

4	200	0.29
5	200	0.92
6	200	2.91
7	200	9.19
8	200	29.06

References

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