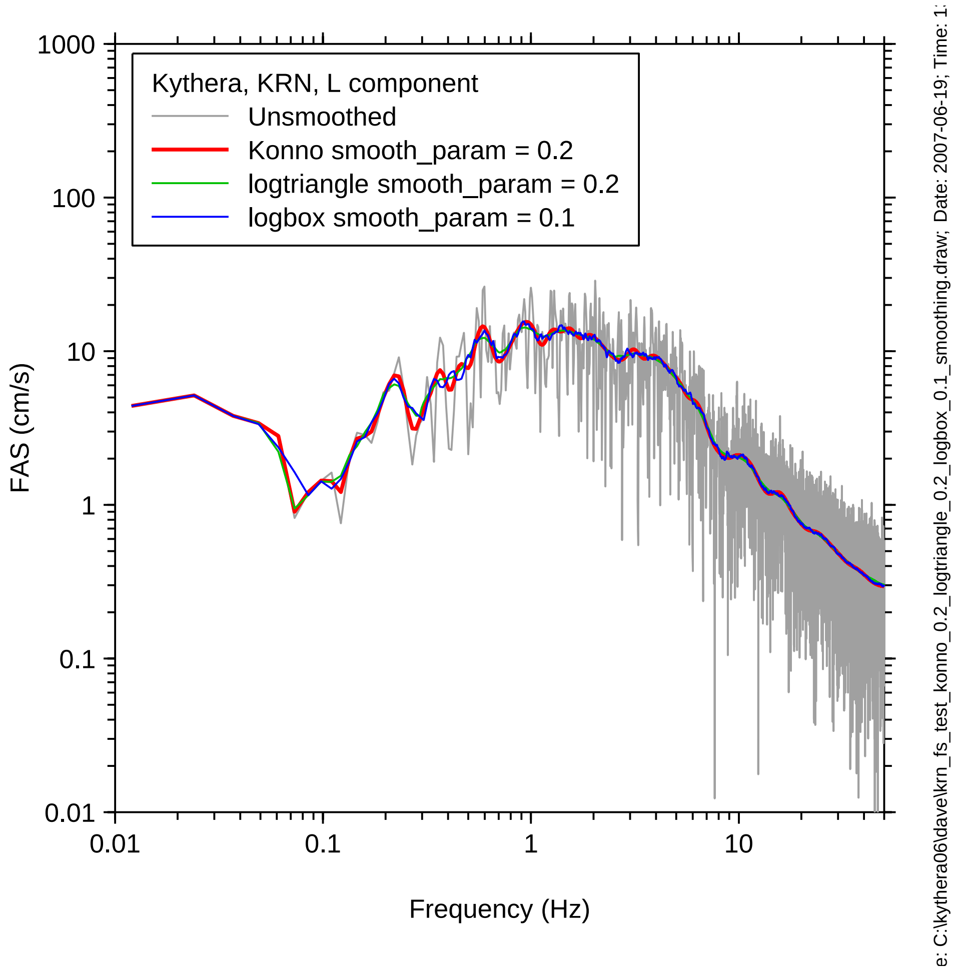
Dave Boore's notes on smoothing over intervals that are constant over logarithmically-spaced frequencies

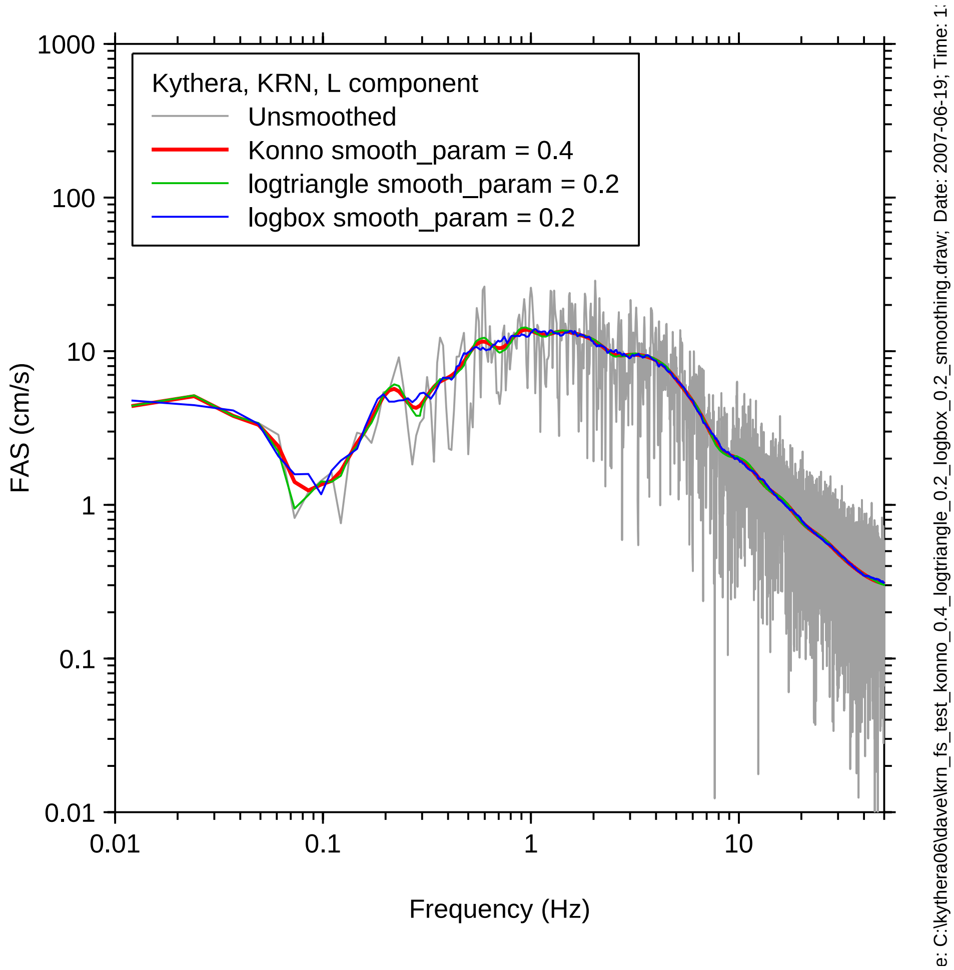
In seismological applications Fourier spectra (FAS) are often plotted using a log scale for the frequency axis (and also for the ordinate axis). It is often desirable to work with a smoothed version of the FAS. FAS from FFTs are defined at equally spaced frequencies, and therefore smoothing using a weighting function that is symmetric when plotted using linear frequency may give what appears to be too much smoothing at low frequencies and not enough smoothing at high frequencies. There may be good technical reasons for choosing a smoothing convolutional operator whose width is constant in log or in linear frequency (e.g., Konno and Ohmachi make such an argument), but I suspect that usually it comes down to subjective judgment: what looks best in the eye of the researcher. I suppose the type of smoothing should depend on why smoothing is being used in the first place. If it is to smooth over the peaks and troughs of a site transfer function, note that for a single layer, the peaks and troughs are equally spaced in frequency, not log frequency (I should look at site response for a multilayered system to see what the "hash" looks like), and in this case smoothing using a function with constant width in linear frequency may be appropriate. On the other hand, if the smoothing is to reduce "noise" in an FAS that will be used to fit a spectral model, it might be more desirable to use a convolutional operator that has a constant width in log frequency.

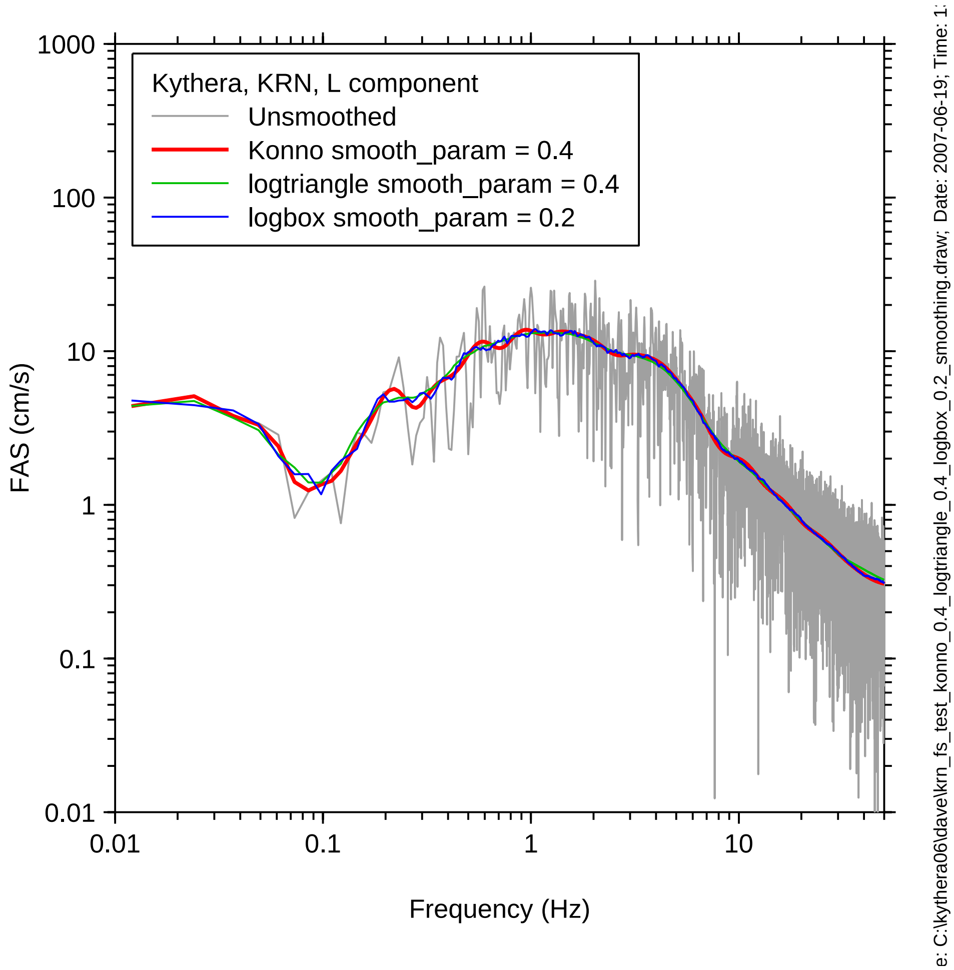
I show here the results of smoothing using three weighting functions: 1) uniform weighting over the interval (a boxcar); 2) triangular weighting over the interval; and 3) the Konno and Ohmachi (1998) $\sin \chi / \chi$ function (using the portion between the zero points on either side of the center frequency---see their equation (4)).

I show a few plots comparing the unsmoothed FAS and the smoothed FAS. I tried to choose smoothing parameters for the three functions that result in similar smoothing.

Here are the results:

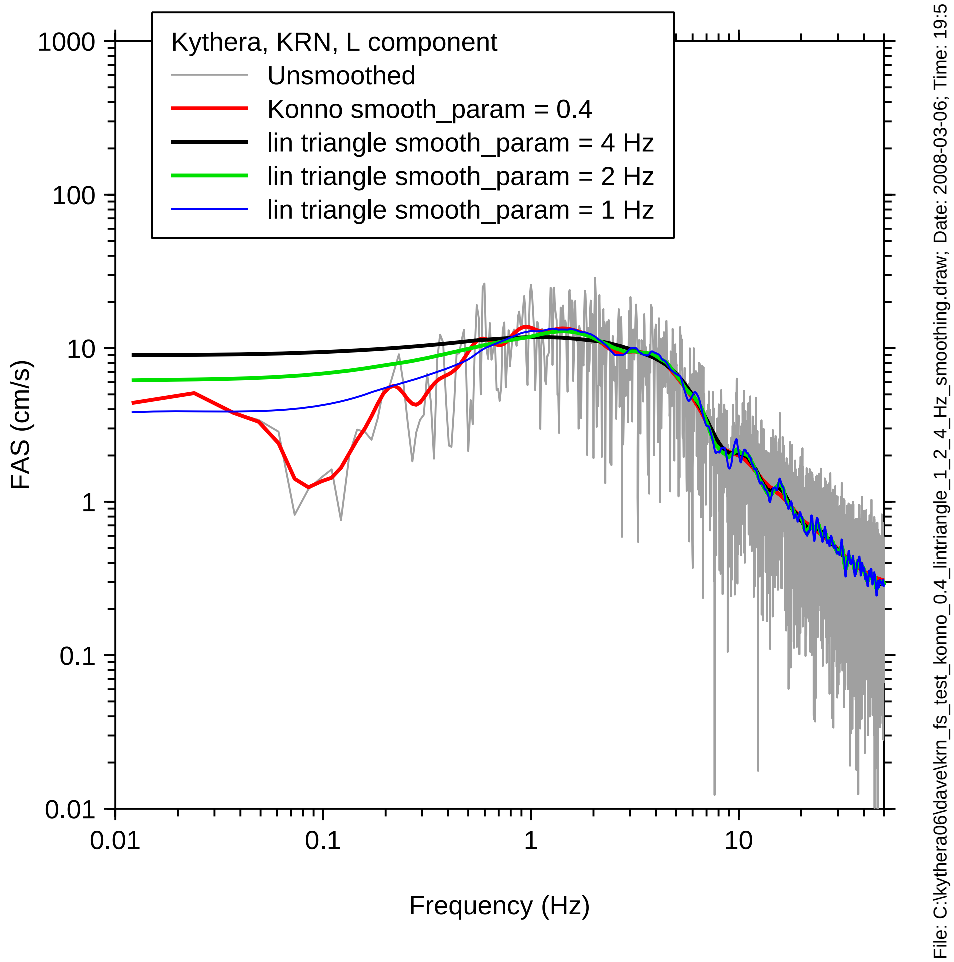

Kythera, KRN, L component
Unsmoothed
Konno smooth_param = 0.2
logtriangle smooth_param = 0.2
logbox smooth_param = 0.1
FAS (cm/s)
1000
100
10
1
0.1
0.01
0.01
0.1
1
10
Frequency (Hz)
e: C:\kythera06\dave\krn_fs_test_konno_0.2_logtriangle_0.2_logbox_0.1_smoothing.draw; Date: 2007-06-19; Time: 1:


Kythera, KRN, L component
Unsmoothed
Konno smooth_param = 0.4
logtriangle smooth_param = 0.2
logbox smooth_param = 0.2
FAS (cm/s)
Frequency (Hz)
1000
100
10
1
0.1
0.01
0.01
0.1
1
10
e: C:\kythera06\dave\krn_fs_test_konno_0.4_logtriangle_0.2_logbox_0.2_smoothing.draw; Date: 2007-06-19; Time: 1:

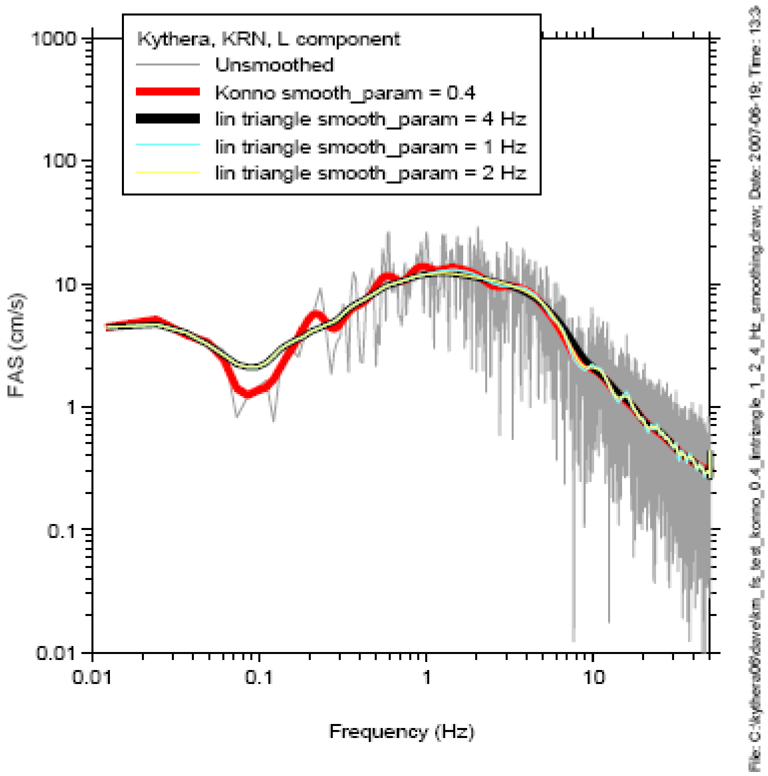


Comment: to my eye, the box function has more chatter than the other two (use the zoom button in Acrobat to better see this), and thus is the least desirable smoother (although in practical applications the difference between the three functions is probably not important). The Konno and Ohmachi smoothing function seems to work well; the smoothing operation is somewhat slower than using the other operators (because of the evaluations of sin and log), but that is probably not relevant for most applications. Note that the Konno and Ohmachi function could be adapted for use when smoothing over a constant frequency interval is desired (simply defining χ in terms of linear *f*).

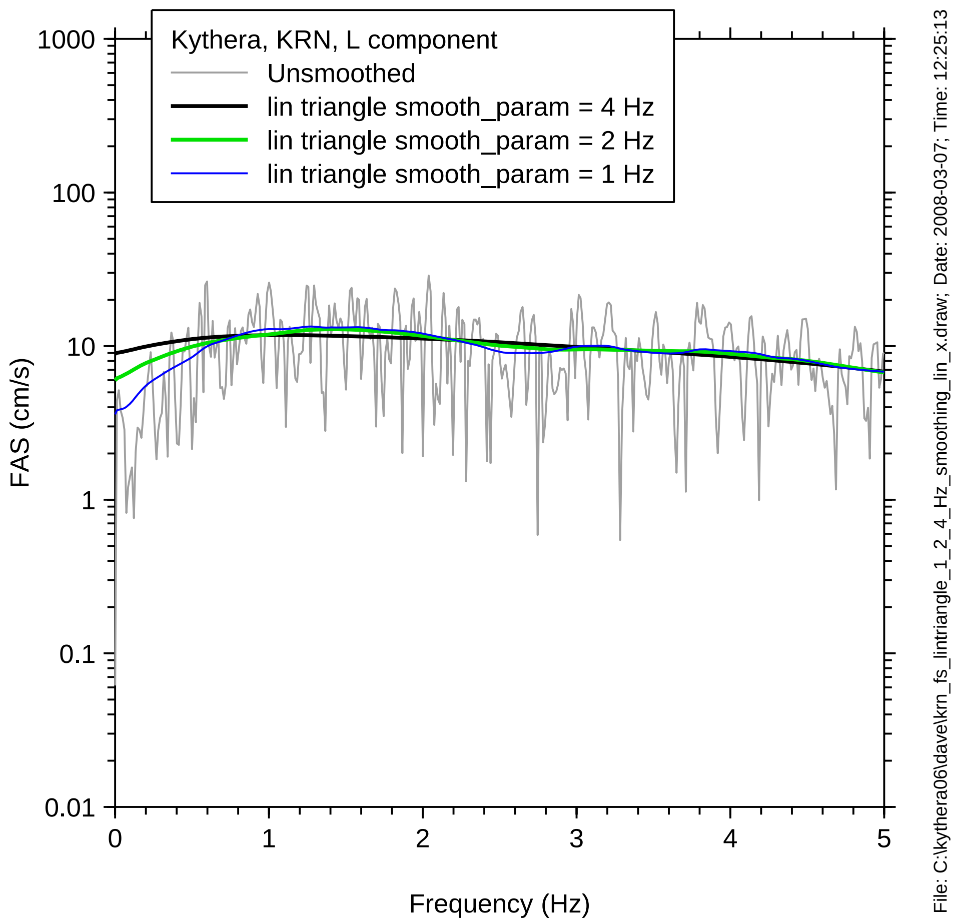
For completeness, here is a figure showing the result of smoothing using an operator whose width is constant in linear frequency (the figure shows the results using three different widths). Note that when plotted using a log abscissa, there appears to be too much smoothing at low frequencies and not enough at high frequencies, compared to a smoothing operator with a constant width in log frequency.



(note: in the July 2007 version of this document the figure above was incorrect for the linear smoothing operators. Here is the earlier version:



And here is the correct figure, plotted using a linear frequency axis extending only to 5 Hz. It is apparent from this figure that the smoothing operators are now working properly. I discovered the error in the previous figure when preparing the TSPP OFR on 06 March 2008, and then I recall realizing sometime after July 2007 that there was a mistake---but I never corrected this document, and I cannot find any plots or notes dealing with this.



References

Konno, K. and T. Ohmachi (1998). Ground-motion characteristics estimated from spectral ratio between horizontal and vertical components of microtremor, *Bull. Seism. Soc. Am.* **88**, 228--241.