Properties of Butterworth Filters as Used in My Record-Processing Software (TSPP)

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Filter Properties:

There is a fundamental difference in the shapes of a causal and acausal Butterworth filter, at least when the acausal filter is computed by filtering the time series twice with a causal filter, running the filter forward and backward through the time series (this produces zero phase distortion which is a defining characteristic of acausal filters; any shape can be used for an acausal filter constructed in the frequency domain).

The response for a causal filter is given by eq. 15.8-6 in Kanasewich (1981):

$$Y = \sqrt{(f/f_c)^{2p} / (1+(f/f_c)^{2p})} . \qquad (1)$$

The acausal filter used in the BAP processing program (Converse and Brady, 1992) and in my processing programs (Boore, 2008) is a time-domain filter, obtained by applying the causal filter twice. The consequence is that the response of the acausal filter is

$$Y = (f/f_c)^{2p} / (1+(f/f_c)^{2p}) . \qquad (2)$$

Note that for the same p as input, for $f/f_c \ll 1$ the causal filter goes as

$$Y \rightarrow (f/f_c)^p$$

whereas the acausal filter falls off more rapidly:

$$Y \rightarrow (f/f_c)^{2p} .$$

Also note the difference when $f = f_c$:

$$\text{causal}: y = 1/\sqrt{2}$$

and

$$\text{acausal}: Y = 1/2 .$$

This is true no matter what value is chosen for p. Because of this, it is not possible to make the response of the causal and acausal filters the same when the filters are applied in the time domain. In particular, caution should be used if p is chosen so as to make the low-frequency asymptotes the same. This requires p for the acausal filter to be half that of the causal filter. The problem is that the response for frequencies higher than f_c will be reduced for the acausal as compared to the causal filter. Designing an acausal filter in the frequency does not have this problem; its response can be made identical to the causal filter response.

The basic parameter in the Converse and Brady (1992) BAP filter routines, used in my record processing software, is $nroll$, where $nroll \equiv 0.5p$. This in itself is confusing (why not use p?). But using either $nroll$ or p leads to another problem: the same value of the order parameter leads to different asymptotic filter behavior, depending on whether the filter is causal or acausal (as shown above). This is confusing, so I now use $nslope$ as the input parameter in my processing program. In this way I can guarantee that the asymptotic behavior is the same for both causal and acausal filters, as follows:

$$Y(f) \rightarrow f^{nslope}$$

for low-cut filters and

$$Y(f) \rightarrow 1/f^{nslope}$$

for high-cut filters. $nroll$ is related to $nslope$ by the following equations:

$$nroll = 0.5 * nslope$$

for causal filters and

$$nroll = 0.25 * nslope$$

for acausal filters. One important constraint on specifying $nslope$ for use in my processing is that

$$nslope = 2, 4, 6, \text{etc.}$$

for causal filters and

$$nslope = 4, 8, 12, \text{etc.}$$

for an acausal filter. This is because the BAP filter subroutines use $nroll$ as the fundamental input parameter, $nroll$ must be an integer, and the equations relating $nslope$ to $nroll$ involve powers of 2 and 4 for causal and acausal filters, respectively.

A word about notation: As shown above, the response of a time-domain acausal filter is not that of a Butterworth filter. Bazzuro et al. (2004) use the terminology "$npnp$", where "n" is the number of poles of standard Butterworth filter, and "p" indicates one application of the Butterworth filter (NOT the order of the Butterworth; it would have been better to use "b" rather than "p"). The "$npnp$" notation is used to indicate that two passes of a Butterworth filter are used. So, for example, "$2p2p$" means that the filter response is equivalent to two passes of a two-pole Butterworth filter in the time domain, with an asymptotic dependence as f^4 (for a low-cut filter). This notation is used to distinguish this case from that in which an acausal filter is simulated in the frequency domain, in which case the filter can have a true Butterworth response function. In this case a filter having the same asymptotic behavior (f^4) would be designated a "$4p$" filter--- a standard 4-pole Butterworth filter (but the values of the filters at $f = f_c$ would differ, the time domain filter having a value of $1/2$ and the frequency domain filter having a value of $1/\sqrt{2}$.

Choice of *nslope*:

I suggest looking at the zeroth-order corrected unfiltered velocity to estimate the order filter needed. A trend given by a polynomial of order n corresponds to a perturbation in acceleration given by a polynomial of order $n-1$, with a Fourier transform going as $1/f^n$ at low frequency (e.g., a linear trend in velocity corresponds to a step in acceleration, with a low-frequency spectrum going as $1/f$). The filter must be chosen such that it decays at low frequencies more quickly than $1/f^n$. Since the low-frequency asymptotic behavior of the low-cut filters in my programs goes as f^{nslope}, this means that $nslope$ should be chosen such that

$$nslope > n$$

subject to the restrictions about the allowed values given earlier. In routine processing I often use the conservative value $nslope = 8$.

References

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