Generalization of 2-Corner Frequency Source Models Used in SMSIM

David M. Boore

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Many of the source spectra models available in SMSIM have two corner frequencies, but only one of these models has the option of varying the high-frequency spectral level, as the other 2-corner models are completely determined by specified relations between the corner frequencies and magnitude (see Tables 2 and 3 in Boore, 2003, for a concise and convenient summary of the various models). In this note I provide equations for generalizing two-corner models to allow the high-frequency source spectral level to be determined by the stress parameter $\Delta\sigma$ (the basic idea being that the 2-corner model will have the same high-frequency source spectral level as a single-corner source model with the specified $\Delta\sigma$). The first model is already in SMSIM (source model 11); the source spectrum is the multiplication of two function of frequency. The second model (source model 12) is new to these notes; it is the summation of two functions of frequency, and as such, it is a generalization of the source spectra used by Atkinson and Boore (1995) and Atkinson and Silva (2000). I first discuss the multiplicative spectrum model, and this is followed by a discussion of the additive spectrum model.

Generalized Multiplicative Source Spectrum

Let the acceleration source spectrum be proportional to:

$$A \propto f^2 \frac{1}{\left[1+\left(f/f_a\right)^{pf_a}\right]^{pd_a}} \frac{1}{\left[1+\left(f/f_b\right)^{pf_b}\right]^{pd_b}} \tag{1}$$

Where " pf " and " pd " stand for "*p*ower of *f*requency" and "*p*ower of *d*enominator". For high frequencies, this becomes

$$A_{HF} \propto \frac{f^2}{f^{(pf_a\times pd_a + pf_b\times pd_b)}} f_a^{\,pf_a\times pd_a} f_b^{\,pf_b\times pd_b} \tag{2}$$

For an ω^2 model, this constraint must be satisfied:

$$pf_a \times pd_a + pf_b \times pd_b = 2 \tag{3}$$

If this constraint is satisfied, then the powers pf and pd can be related to an equivalent stress parameter and single corner frequency model, as follows. For a single corner frequency model with corner frequency f_c, the high-frequency spectral level goes as

$$A_{HF} \propto f_c^{\,2} \tag{4}$$

Equating (2) and (4), with the constraint (3) gives

$$f_c^{\,2} = f_a^{\,pf_a \times pd_a} f_b^{\,pf_b \times pd_b} \tag{5}$$

The procedure then is to use the relation

$$f_c = 4.906 \times 10^6 \beta \left(\Delta\sigma / M_0 \right)^{1/3} \tag{6}$$

to obtain f_c given $\Delta\sigma$ and M_0, where M_0 comes from moment magnitude **M** using the relation

$$\log M_0 = 1.5\mathbf{M} + 16.05\,. \tag{7}$$

Assuming that f_a is specified by the user, in the following way:

$$\log f_a = c_{1fa} + c_{2fa} \left(\mathrm{M} - M_{fa} \right), \tag{8}$$

then equation (5) can be used to find f_b:

$$f_b = \left[\frac{f_c^{\,2}}{f_a^{\,pf_a \times pd_a}} \right]^{1/(pf_b \times pd_b)} \tag{9}$$

I illustrate this model for two sets of the powers, both satisfying the constraint in equation (3). In the first example, $pf_a = pf_b = 2$ and $pd_a = pd_b = 0.5$. Figure 1 shows the source spectra for this model, assuming M 6 and $\Delta\sigma = 100$ bars, for a series of f_a.

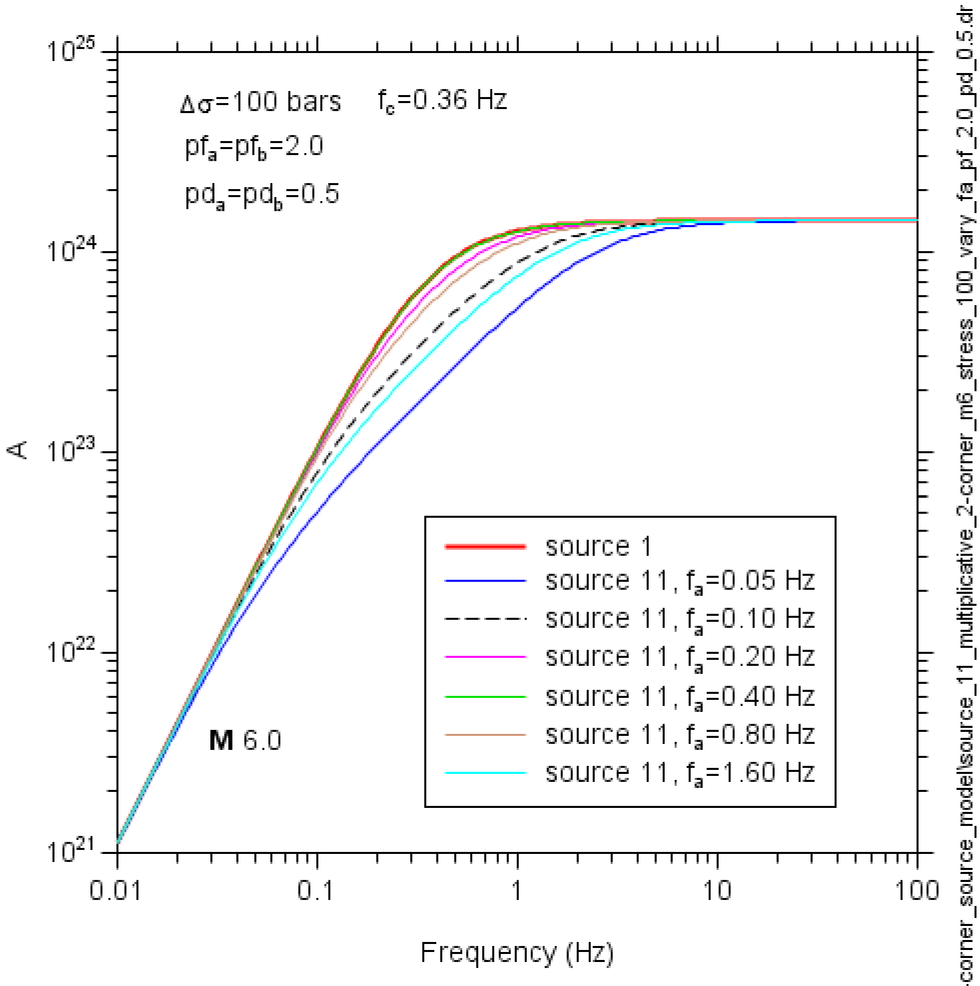

Figure 1.

Note that with these choices of the powers, the two-corner sources merge into the single corner model when $f_a = f_c = 0.36$ Hz, as expected from the formulation above. In contrast, Figure 2 shows the source spectra for $pf_a = pf_b = pd_a = pd_b = 1.0$, and for this case, the 2-corner model never approaches the one-corner model.

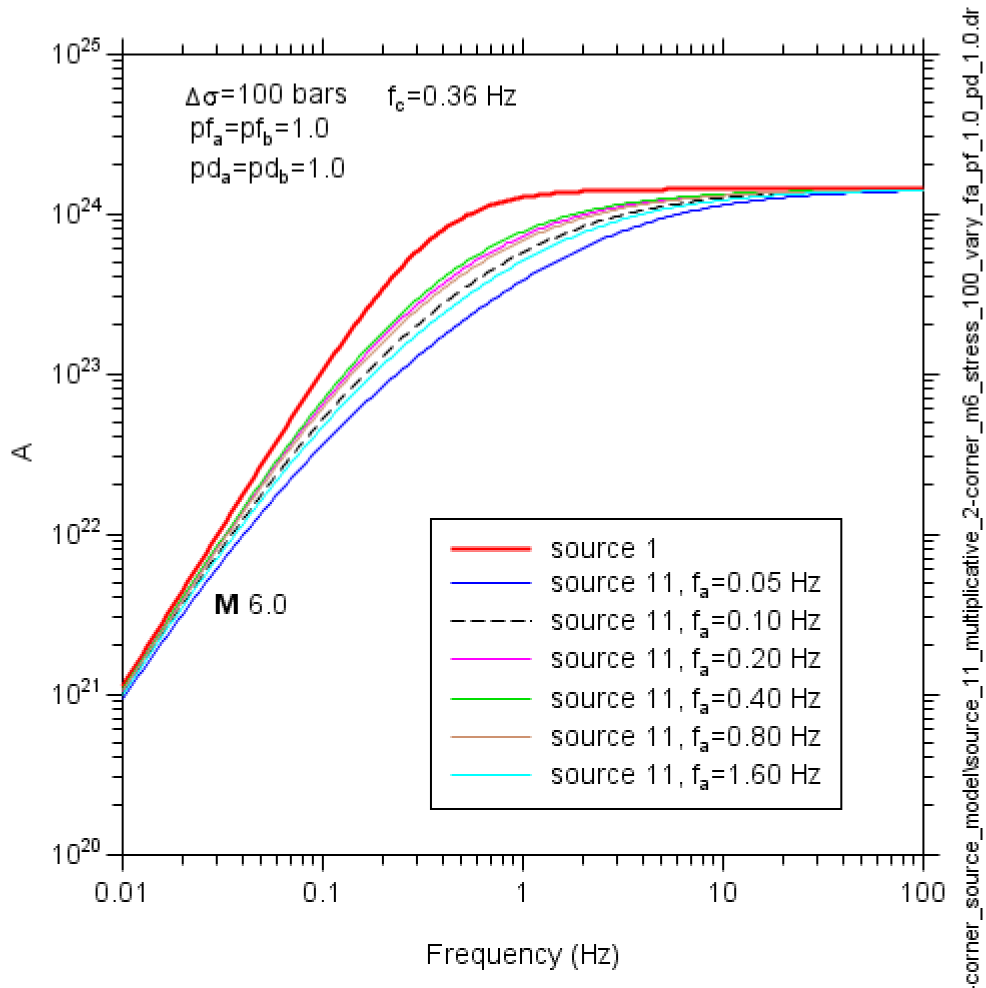

Figure 2.

Generalized Additive Source Spectrum

Let the acceleration source spectrum be proportional to:

$$A \propto f^2 \frac{1-\varepsilon}{\left[1+\left(f/f_a\right)^{pf_a}\right]^{pd_a}} + \frac{\varepsilon}{\left[1+\left(f/f_b\right)^{pf_b}\right]^{pd_b}} \tag{10}$$

where " pf " and " pd " stand for "*p*ower of *f*requency" and "*p*ower of *d*enominator". I proposed this source model to Gail Atkinson in a 1992 personal communication, and she used it to derive a source spectral model for ENA earthquakes (Atkinson, 1993), and this form of the source model was subsequently used in other papers by Gail and her colleagues. For high frequencies, this becomes

$$A_{HF} \propto \left(\frac{f^2}{f^{pf_a \times pd_a}}\right)(1-\varepsilon) f_a^{pf_a \times pd_a} + \left(\frac{f^2}{f^{pf_b \times pd_b}}\right) \varepsilon f_b^{pf_b \times pd_b} \tag{11}$$

For an ω^2 model, the following constraint must be satisfied:

$$pf_a \times pd_a = pf_b \times pd_b = 2 \tag{12}$$

and the high-frequency level is:

$$A_{HF} \propto (1-\varepsilon) f_a^2 + \varepsilon f_b^2 \tag{13}$$

If the constraint in equation (12) is satisfied, then given $\varepsilon(\mathbf{M})$ and $f_a(\mathbf{M})$, $f_b(\mathbf{M})$ can be determined by equating the high-frequency source spectral level to the level for a single corner frequency model, as follows. This then generalizes the additive two-corner model by letting the high-frequency level by determined by a stress parameter $\Delta\sigma$ (although there may be some constraints on $\varepsilon(\mathbf{M})$ and $f_a(\mathbf{M})$ in order for the resulting source spectrum to make sense, such as f_b being a real number---I need to do some exploration of this).

For a single corner frequency model with corner frequency f_c, the high-frequency spectral level goes as

$$A_{HF} \propto f_c^2 \tag{14}$$

Equating (13) and (14) gives

$$f_b = \sqrt{\frac{f_c^2 - (1-\varepsilon) f_a^2}{\varepsilon}} \tag{15}$$

The procedure now is to use the relation

$$f_c = 4.906 \times 10^6 \beta \left(\Delta\sigma / M_0\right)^{1/3} \qquad \text{(equation 6, repeated)}$$

to obtain f_c given $\Delta\sigma$ and M_0.

Assuming that f_a and ε are specified by the user, such as in the following ways

$$\log f_a = c_{1fa} + c_{2fa}\left(\mathbf{M} - \mathbf{M}_{fa}\right) \tag{16}$$

and

$$\log \varepsilon = c_{1\varepsilon} + c_{2\varepsilon}\left(\mathbf{M} - \mathbf{M}_{\varepsilon}\right) \tag{17}$$

then equation (15) can be used to find f_b.

Note that for the f_a and $\varepsilon < 1$ for a given **M** that there will be a value of $\Delta\sigma$ below which f_b is not defined. This occurs when the numerator under the radical in equation (15) equals 0.0. From equations (6) and (15), the lower limit for $\Delta\sigma$ is

$$\Delta\sigma = \left[\frac{\sqrt{1-\varepsilon}}{\xi} f_a \right]^3 M_0 \tag{18}$$

where

$$\xi = 4.906 \times 10^6 \beta \tag{19}$$

I have revised the SMSIM programs to include source model 12 (source model 11 was already in the programs). This required a change to the params files, because the coefficients of equation (17) must be included in the params file.

I tested the revision by using Atkinson and Silva's (2000) equations for f_a and ε. Figure 3 shows the source spectra for the additive model for several values of M and $\Delta\sigma$, compared to the single-corner source spectra (to check that the 2-corner spectra have the same high-frequency levels as the single-corner spectra). I used $pf_a = pf_b = 2.0$ and $pd_a = pd_b = 1.0$ for the example in this and subsequent figures.

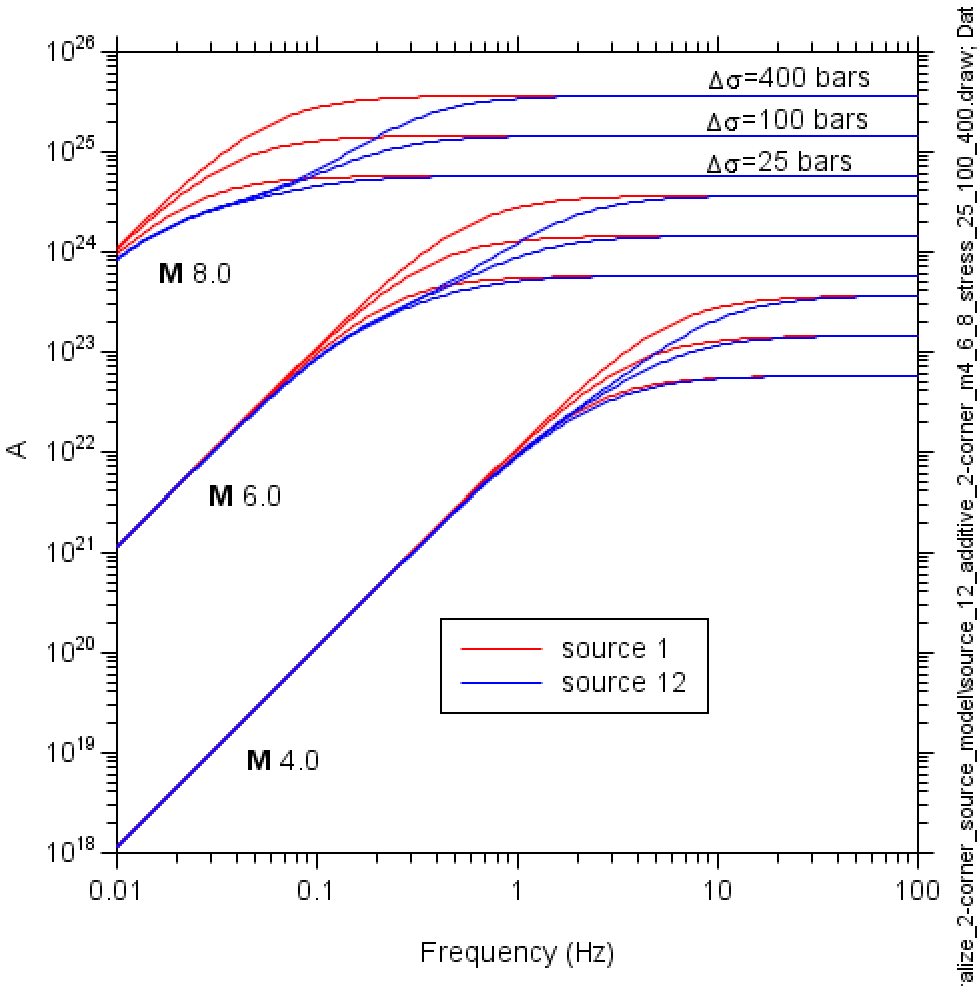

Figure 3. Source spectra

Figure 4 shows the source spectra for a suite of ε values differing by a factor of 2, and ranging from 0.01 to 0.64, for a specified value of f_a.

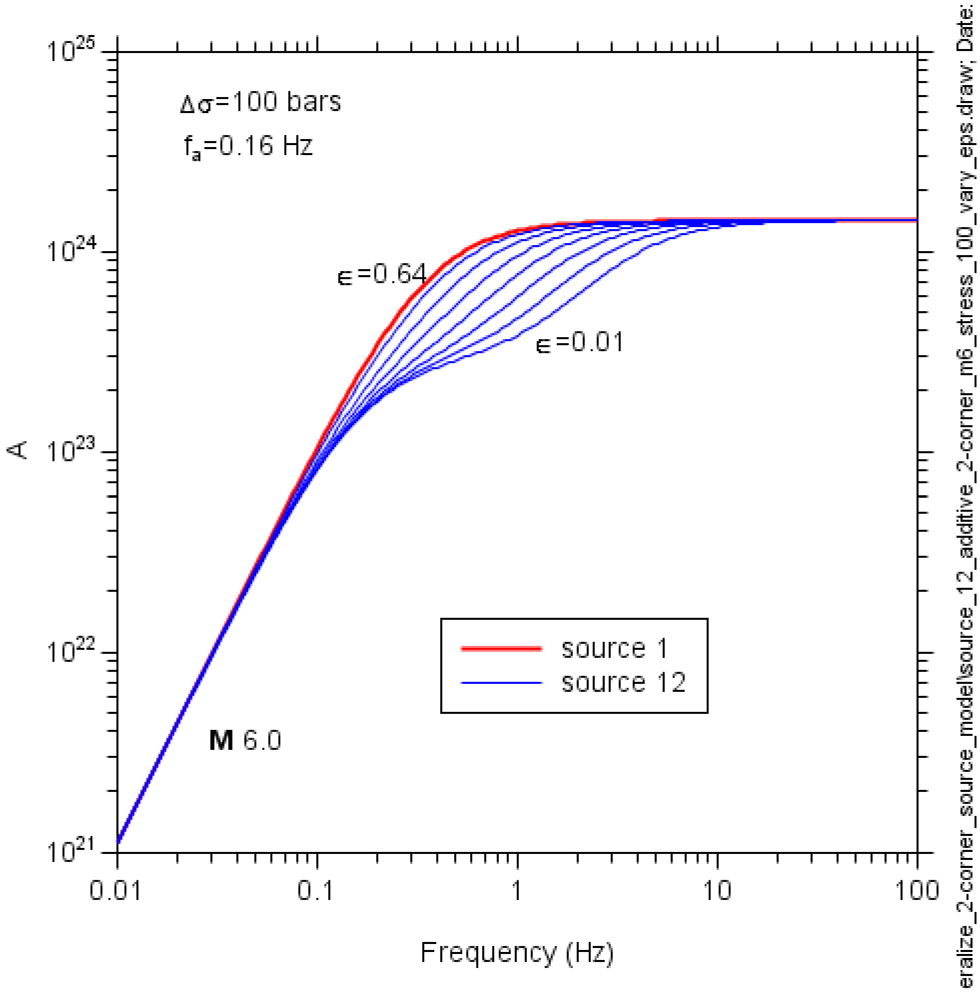

Figure 4.

I simulated the motions at 10 km for a M 6 earthquake, with an effective single-corner stress parameter of 400 bars. Figure 5 shows the Fourier spectrum for the 1- and 2-corner models for this case, computed from the basic equations for the spectra (what is called "model" in the figure), from an average of 100 time-domain simulations, and the spectrum from the last of those simulations. The steep decay of the Fourier spectrum at low frequency is due to the inclusion of a low-cut filter with a corner frequency of 0.04 Hz, decaying as f^8 at low frequency.

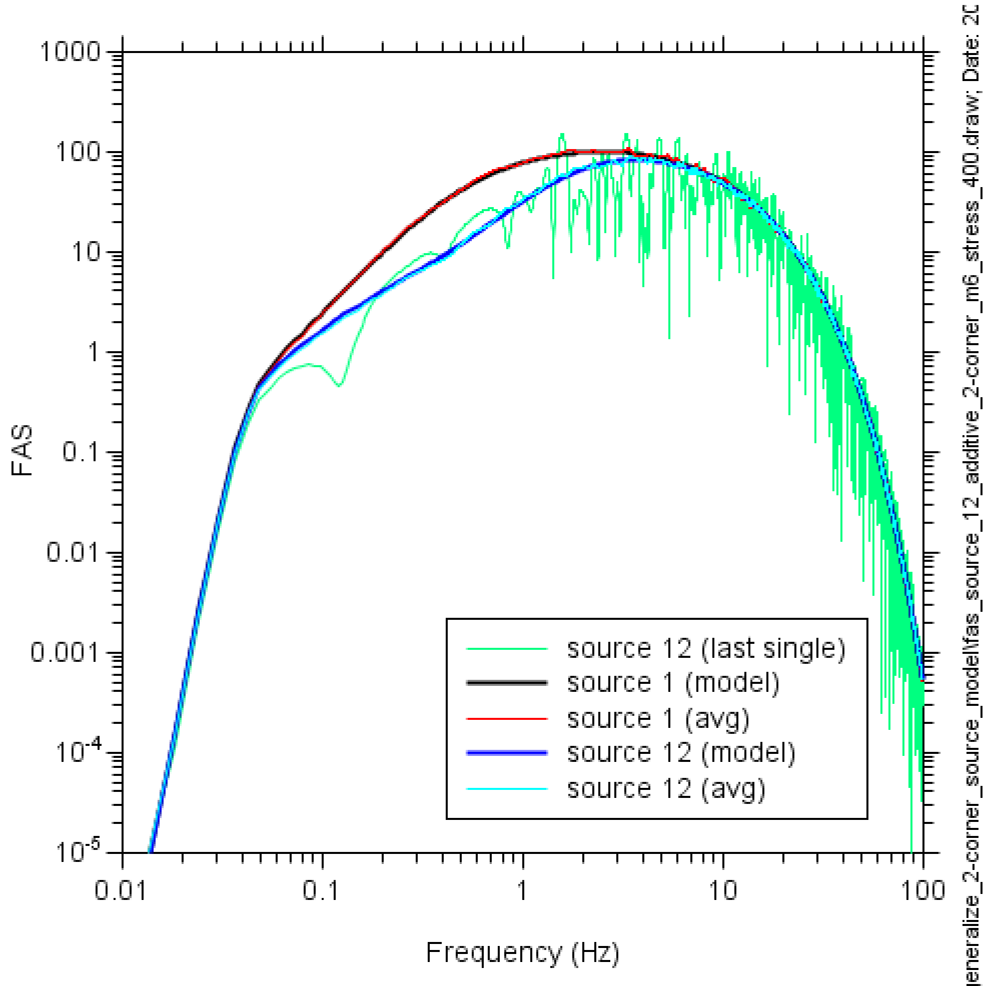

Figure 5. Fourier acceleration spectra at R=10 km, **M** 6, $\kappa = 0.04$ s, and generic western rock crustal amplifications and *Q*.

The response spectra are shown in Figure 6, computed for the 1- and 2-corner source models, using both random-vibration and time-domain (with 100 simulations) computations.

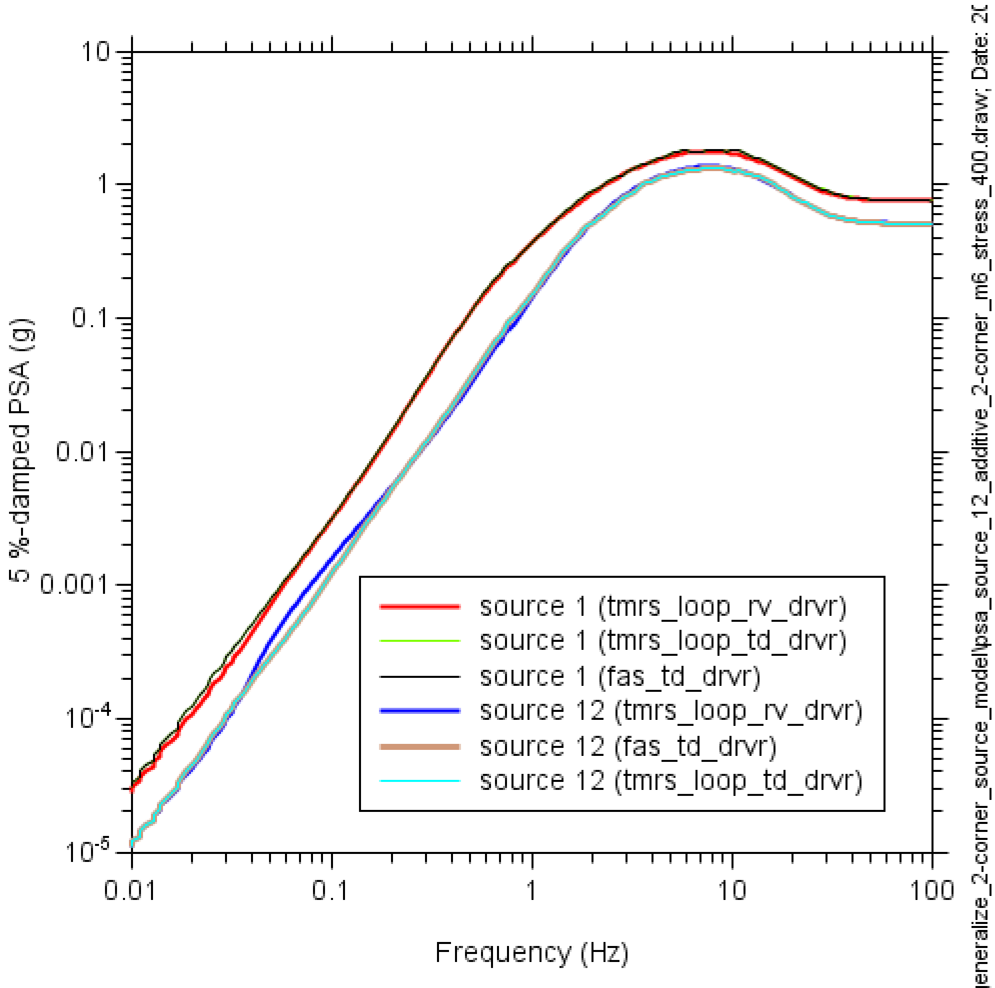

Figure 6. Response spectra for cases discussed in Figure 5 (the jitter in the curves for low frequencies is due to the frequency in the output files not having enough resolution).

Note that the TD and RV simulations are in close agreement, except for frequencies between about 0.04 and 0.10 Hz. I'm not sure of the reason for this disagreement, but it might have to do with the oscillator adjustments being used in the RV simulations (I am using those of Boore and Thompson, 2012). As usual, it is always a good idea to check RV results with TD computations.

The difference in amplitudes of the 1- and 2-corner (source 1 and source 12) PSA for frequencies on either side of the region of the source 12 sag (about 0.04 to 3 Hz, as shown in Figure 3) is primarily a result of how the source durations are computed. For source 1 the source duration equals $1/f_c$, while for the 2-corner source model (source 12), the source duration is given by

$0.5/f_a + 0.5/f_b$. The result is that the source duration equals 1.77 and 4.24 for sources 1 and 12, respectively, and this will result in a difference in the rms acceleration of $\sqrt{4.24/1.77} = 1.5$, with the source 12 rms acceleration being smaller than the source 1 rms acceleration. The PSA in Figure 6 at high frequency, which are equal to peak acceleration, differ by a factor of 1.5, as expected from the ratio of durations.

As an aside, the equation above for the two-corner source duration differs from that used in Atkinson and Boore (1995) and Atkinson and Silva (2000): $0.5/f_a + 0.0/f_b$. The problem with this source duration is that it leads to a discontinuity at the magnitude for which the two corner frequencies become equal. I prefer using equal weights of 0.5 for both inverse corner frequencies, as this avoids the discontinuity. In addition, as magnitude increases f_b generally increases much more rapidly than f_a, and as a result the duration is primarily controlled by the term $0.5/f_a$, which is the equation used by Atkinson and Boore (1995) and Atkinson and Silva (2000).

Discussion

The two generalized 2-corner source models are able to approximate a wide range of spectral shapes, as indicated in Figures 1, 2, and 4. This flexibility comes at the expense of a number of coefficients that must be specified for any application. I have not worked with the models (and data to which the models can be applied) enough to make strong recommendations for these coefficients, but at this time, I suggest the following: for source 11 (the multiplicative model), $pf_a = pf_b = 2.0$ and $pd_a = pd_b = 0.5$, with f_a to be determined by fitting data (or GMPEs, or finite-fault simulations); for source 12 (the additive model), $pf_a = pf_b = 2.0$ and $pd_a = pd_b = 1.0$, f_a from Atkinson and Boore (1995) or Atkinson and Silva (2000), and ε to be determined by fitting data (or GMPEs, or finite-fault simulations).

References

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