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LETTERS TO THE EDITOR

A NOTE ON THE USE OF RANDOM VIBRATION THEORY TO PREDICT PEAK AMPLITUDES OF TRANSIENT SIGNALS

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ABSTRACT

Random vibration theory offers an elegant and efficient way of predicting peak motions from a knowledge of the spectra of radiated energy. One limitation to applications in seismology is the assumption of stationarity used in the derivation of standard random vibration theory. This note provides a scheme that allows the standard theory to be applied to the transient signals common in seismology. This scheme is particularly necessary for predictions of peak response of long-period oscillators driven by short-duration ground motions.

INTRODUCTION

Predicting peak time-domain amplitudes that correspond to a given amplitude spectrum is a common problem in seismology. For example, McGuire and Hanks (1980), Hanks and McGuire (1981), Boore (1983), Joyner (1984), and McGuire *et al.* (1984) have used various seismological models of the radiated spectrum to predict parameters of strong ground motion, such as peak acceleration, velocity, and response spectra. Although the peak motions can be obtained from time-domain simulations, this process can be cumbersome and expensive. A more efficient method that is particularly appropriate if the waveforms have a random character is to use some results from random vibration theory (RVT) that relate the expected peak amplitude [$E(y_{\max})$] out of a set of N amplitudes to the rms of the time series (y_{rms}) and thus, by using Parseval's theorem, to the spectrum (a detailed discussion is contained in Boore, 1983). From the RVT results of Cartwright and Longuet-Higgins (1956)

$$E(y_{\max})/y_{rms} = f(N) \tag{1}$$

where $f(N) \approx \sqrt{2 \ln N}$ when N is large. The RVT assumes stationary time series. Very often, however, the time series of seismological interest are far from being stationary. In spite of this, Boore (1983) showed that RVT gave good predictions of peak acceleration and velocity determined from time-domain simulations. Some difficulties were encountered, however, for predictions of the peak response of long-period, lightly damped oscillators. The purpose of this note is to discuss a scheme that to a large extent overcomes these difficulties, at least for damping of 5 per cent and greater.

ANALYSIS

The essence of the problems encountered lies in defining time-domain durations. Duration enters in two ways: in determining N and in calculating y_{rms}. The number of extrema, N, that might produce the peak motion is given by a characteristic frequency of the motion times the duration of quasi-stationary shaking. The characteristic frequency is given by ratios of spectral moments [e.g., equations (26) and (27) in Boore (1983)] and in most applications the duration of shaking is taken to be the source duration (D_s). The duration (D_{rms}) used in computing the rms is more difficult to determine. At first glance, from the definition of rms it would seem that the duration should be D_s. From Parseval's theorem, we would then have

$$y_{rms} = \left[\frac{1}{D_s} \int_{-\infty}^{\infty} |Y|^2 \, df\right]^{1/2} \tag{2}$$

where $|Y|$ is the amplitude spectrum corresponding to a seismological model. The problem with using this equation is that in oscillators, the energy content in the spectrum can be distributed beyond D_s, and therefore equation (2) will give a false estimate of y_{rms} for the actual response. The motion beyond D_s, however, cannot be used in determining N, because the response will have a steady decay after the excitation stops and cannot produce the peak motion. Two durations are needed, therefore, in using RVT to predict the peak response of an oscillator subjected to transient excitations: one to use in estimating N and one to use in determining y_{rms}. In our scheme, which is similar to that reported by Boore (1983), N is determined from D_s (usually given as the inverse corner frequency in simple source-scaling models), with the constraint that N be greater than or equal to 2. The determination of D_{rms} is the main contribution of this note. Our first idea was to set D_{rms} equal to the sum of the source duration and the oscillator decay time. For long-period oscillators excited by short duration motions, however, this scheme can lead to grossly exaggerated estimates of D_{rms}. As an example, Figure 1 shows the response of a 6-sec, 5 per cent-damped oscillator to $M = 4$ and $M = 7$ earthquakes at 10 km distance (for ease of comparison, the model parameters used in the illustrations in this paper are the same as those used in constructing Figure 15 of Boore, 1983). Clearly, the D_{rms} for the smaller earthquake should be much shorter than for the larger event; if our first approach is used, however, D_{rms} would have been 20.0 sec and 29.4 sec for the two events. From considerations of oscillator response to excitations much longer and much shorter than the oscillator period, we finally settled on the following equation for determining D_{rms}

$$D_{rms} = D_s + D_o\left(\frac{\gamma^n}{\gamma^n + \alpha}\right) \tag{3}$$

where α and n are adjustable parameters, $\gamma = D_s/T_o$ and T_o is the oscillator natural period. The oscillator duration, D_o, is given by

$$D_o = T_o/2\pi\zeta \tag{4}$$

where ζ is the fractional damping of the oscillator. The form of equation (3) was chosen so that D_{rms} approaches D_s and $(D_s + D_o)$ for excitations that are shorter and longer than the oscillator period, respectively. The adjustable parameters n and α were determined by comparison of RVT and time-domain simulations. Some of these comparisons are shown in Figures 2 and 3, which show the response spectra for the two earthquakes used to generate the motions in Figure 1. The solid lines are based on the average of an ensemble of time-domain simulations (one member of which is shown in Figure 1). The RVT results using various schemes for the choice of D_{rms} are shown by the symbols. From these and similar comparisons not shown here, the provisional values

$$n = 3 \tag{5a}$$

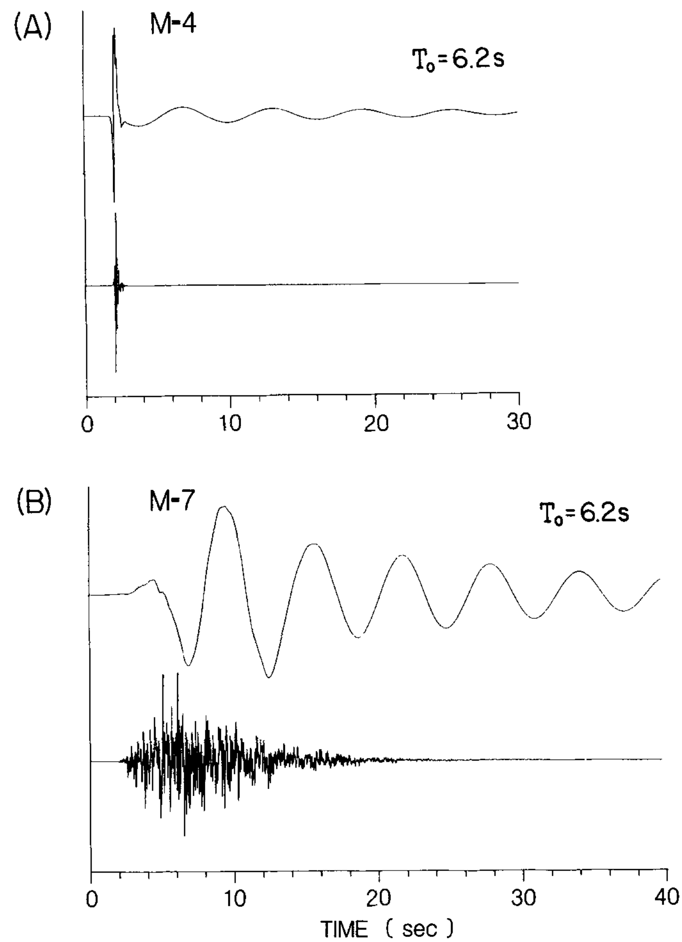

FIG. 1. Response of 6.2-sec, 5 per cent-damped oscillator (*top trace* of each pair) to earthquakes with moment magnitudes of 4 and 7 at 10 km distance. The excitation is shown below the oscillator output. Only relative shape is of interest, the maximum excursion of each trace is scaled individually to a given size. The source durations (D_s) are 0.31 and 9.7 sec for the magnitude 4 and 7 events, respectively, and the oscillator duration (D_o) is 19.6 sec.

and

$$\alpha = \tfrac{1}{3}. \tag{5b}$$

have been determined.

DISCUSSION

Although the scheme described above is simple and not laboriously optimized, it does reconcile the results of RVT and time-domain simulations over the range of magnitude and oscillator frequency of practical interest, thereby permitting the use of the more efficient and elegant RVT methods in predicting peak oscillator response. One of the reasons for its greater efficiency is that RVT returns an estimate of the expected value of the peak parameter without having to do a large series of time-domain simulations. Published applications up to now have included the computation of peak acceleration and velocity, response spectra, seismoscope response, and Wood-Anderson instrument response for earthquakes at local and regional distances (Hanks and McGuire, 1981; Boore, 1983, 1984; Hanks and Boore, 1984; Joyner, 1984).

The success of using RVT in conjunction with the spectral scaling models to predict various measures of strong ground motion suggests that predictions of short-period teleseismic motion can also be made. Changes in the existing computer codes to do this are straightforward. Because the RVT method is largely based in the

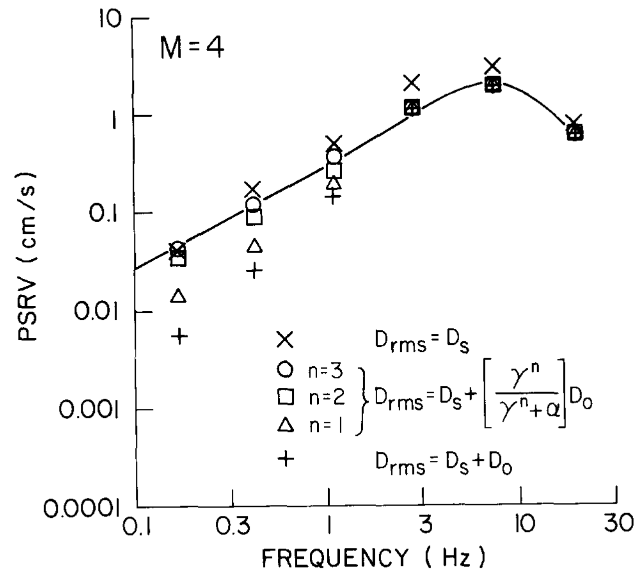

FIG. 2. Comparison of pseudo-velocity response spectrum (PSRV) computed from time-domain simulations (solid line) and random vibration theory for a magnitude 4 earthquake (compare with Figure 15 in Boore, 1983). A sample accelerogram for the earthquake and the corresponding response of the lowest frequency oscillator used in this figure are shown in Figure 1A. The RVT results are for a range of n, with $\alpha = \frac{1}{3}$. Clearly, $n = 3$ is preferred.

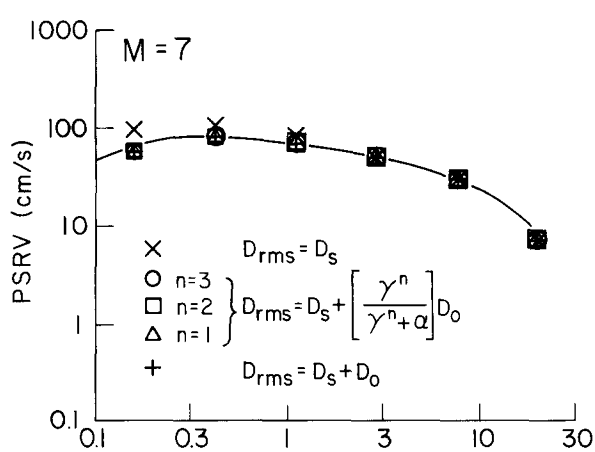

FIG. 3. Same as Figure 2, but for a magnitude 7 earthquake. A sample accelerogram and oscillator response are shown in Figure 1B. Because the excitation is longer than the longest period oscillator considered, the RVT results are not sensitive to the parameter n.

frequency domain, inclusion of instrument response and anelastic attenuation (with frequency dependence, if necessary) is particularly easy. An obvious application is to studies of seismic scaling laws, in which various earthquake magnitudes are computed from theoretical spectra for comparison with data. Previous studies, such as those of Aki (1967, 1972), Kanamori and Anderson (1975), Geller (1976), and

Nuttli (1983), have considered only the relative change in the peak responses of various instruments rather than the absolute values. Using the RVT method, absolute values can be predicted. Furthermore, to avoid having to synthesize time-domain motions in order to pick off peak values, and thus magnitudes, these studies have commonly assumed that the magnitudes are directly proportional to the logarithms of spectral amplitudes at fixed periods (1 sec for m_b and 20 sec for M_S). Although this assumption may be valid for M_S up to about $7\frac{1}{2}$, Hanks (1979) pointed out that it is questionable for m_b [this assumption was used for m_b be Geller (1976) and Nuttli (1983)]. Dependence on this assumption can be avoided with RVT. In its present form, RVT is capable of predicting the peak response to body-wave excitation. Application to dispersed surface-wave excitation will require further extensions of the method, similar in spirit to those discussed in this note.

A number of improvements in RVT have been made since the publication of the Cartwright and Longuet-Higgins (1956) theory used here (e.g., Mason and Iwan, 1983). Although the Cartwright and Longuet-Higgins theory gives quite satisfactory predictions of peak motions, it may be that further advances will be possible using recent developments in RVT.

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