Bulletin of the Seismological Society of America, Vol. 79, No. 6, pp. 1984–1988, December 1989

THE EFFECT OF DIRECTIVITY ON THE STRESS PARAMETER DETERMINED FROM GROUND MOTION OBSERVATIONS

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A key parameter in the stochastic model for predicting ground motions (Hanks and McGuire, 1981; Boore, 1983) is the stress parameter used in relating the corner frequency to the seismic moment. Because the high-frequency spectral level of an ω-squared model goes as the stress parameter to the 2/3 power, the ground motions at frequencies above the corner frequency are particularly sensitive to this parameter. (Boore, 1983, shows that log $pga \; \alpha \; 0.8 \log \Delta\sigma$, where pga is the peak ground acceleration and $\Delta\sigma$ is the stress parameter.) In practice, the stress parameter is determined by fitting observed ground motions, including peak acceleration, velocity, and response spectra, to the theoretical predictions. The observations are conveniently summarized by the equations obtained from regression of the ground motion data against magnitude and distance (see Joyner and Boore, 1988, for a recent review). As such, the regressions represent an average of the ground motion data for many take-off angles and azimuths from the earthquake source. To the extent that rupture propagation over the fault plane has influenced the spectra, the derived stress parameter has incorporated this directivity effect in some average way. The stochastic models in current use (e. g., Boore and Atkinson, 1987; Silva and Lee, 1987; Toro and McGuire, 1987) represent the source as a point, with no consideration of rupture over the fault plane. This can lead to different estimates of the stress parameter for the same set of earthquakes, depending on the type of waves used and their take-off angles from the source.

To illustrate the problem, consider a set of dynamically similar earthquakes with rupture along the fault strike. The directivity will most strongly affect S waves leaving the source almost horizontally; the effect on teleseismic recordings of P waves will be minor. Thus without explicit consideration of directivity, it is likely that the estimates of the stress parameter from teleseismic P waves would be systematically different from those made using strong-motion recordings of the same set of earthquakes at near distances. This might explain Boore's (1986) finding that strong-motion S-wave data from the western United States led to a stress parameter of 50 bars, while teleseismic P-wave data from a globally distributed set of earthquakes were best fit by a stress parameter closer to 30 bars (although other explanations can be invoked, including regional variations in the stress parameter).

The effect of directivity not only can produce systematic variations in estimates of a parameter related to the physics of the faulting process, it can also lead to errors in modeling of motions from an extended fault if the stress parameter obtained from the point-source stochastic model were used in the extended-fault model (such as that of Joyner *et al.*, 1988). The effect of directivity would be included twice: implicitly in the azimuthally averaged ground motion data, and explicitly in the finite-fault calculations.

In this note, we illustrate the effects of directivity on the derived stress parameter for two models of directivity. We do not intend the results in this note to be the definitive treatment of directivity; rather, our primary purpose in writing this note will be served if it draws attention to a shortcoming of the standard stochastic model and stimulates discussion and further work on the problem.

ANALYSIS

To quantify the arguments above, we consider a point-source model, with modifications to the spectrum that incorporate the effect of directivity. Assuming a unidirectional rupture, we generate the spectra for randomly chosen azimuths and take-off angles, and find the average of the logarithm of the high-frequency spectral levels (which are more strongly affected by directivity than are the spectra at lower frequencies). The angles are chosen from a distribution function such that the resulting average is equal to an average over a specified portion of the focal sphere (e. g., if 0° is downward vertical a range of take-off angles from 120° to 180° represents observations at close distances). Details regarding this Monte Carlo method for finding an average over the focal sphere are in Boore and Boatwright (1984).

For an ω-squared source model, the high-frequency acceleration spectral amplitude in the usual stochastic models varies as

$$|A| \propto M_0^{1/3} \Delta\sigma^{2/3}, \tag{1}$$

where the seismic moment (M_0) and the stress parameter ($\Delta\sigma$) are the fundamental parameters controlling the level of ground motion (e. g., Boore, 1983, equations (3) and (5)). The effect of directivity will generally be to modulate $|A|$ with a function depending on the relative directions of rupture propagation and propagation of waves from the source. We simulate the directivity effect through a factor D, such that equation (1) is modified to

$$|A| \propto M_0^{1/3} \Delta\sigma^{2/3} D^{\gamma} \tag{2}$$

where D is the directivity factor, commonly given for a line source by

$$D = 1/[1 - (v/c)\hat{\mathbf{i}}_{\text{rup}} \cdot \hat{\mathbf{i}}_{\text{ray}}]. \tag{3}$$

In this equation, $\hat{\mathbf{i}}_{\mathbf{rup}}$ is a unit vector in the direction of rupture propagation and $\hat{\mathbf{i}}_{\mathbf{ray}}$ is a unit vector giving the direction in which a ray leaves the source. v and c are the rupture and wave propagation velocities, respectively.

The factor γ in equation (2) is model dependent. We consider two values: $\gamma = 1.0$, following a suggestion by J. Boatwright (personal comm., 1989), and $\gamma = 1.5$. The latter value comes from Joyner's (1984) source model, with the assumption that the lower-frequency of the two corner frequencies in his model is modified by multiplying by the directivity factor D. A similar assumption for the standard one-corner-frequency model gives $\gamma = 2.0$.

The difference in the average high-frequency levels with and without directivity (the latter represented by the subscript "ND") is given by

$$\langle \log|A| \rangle - \langle \log|A|_{\text{ND}} \rangle = \langle \log D^{\gamma} \rangle, \tag{4}$$

where the notation "<>" implies an average of the quantity within the brackets. The difference in the spectral levels can be represented by using equations (1) and (2) to define an equivalent stress parameter ($\Delta\sigma_e$) as follows:

$$\Delta\sigma_e = \Delta\sigma 10^{1.5\langle \log D^{\gamma} \rangle}. \tag{5}$$

In this equation,, if $\Delta\sigma$ is thought of as the actual stress parameter for a set of similar earthquakes, then $\Delta\sigma_e$ is the equivalent stress parameter that would be derived from the data from this set of events if the standard stochastic, point-source model (i. e., equation (1)) were used. The ratio of the two stress parameters ($\Delta\sigma_e/\Delta\sigma$) has been calculated for a range of rupture velocities and three directions of rupture propagation along a fault dipping 45°. (For propagation in the horizontal direction, the results are independent of fault dip.) As mentioned above, two values of γ were used. The ranges of take-off angles were taken to simulate ground motion observations at regional and close distances (60° to 120° and 120° to 180°, respectively). The results are given in Table 1.

Discussion

It is clear from Table 1 that the directivity can strongly influence the stress parameter derived from recordings of earthquakes. Even though the directivity factor D can be less than 1, the average over the focal sphere always produces an equivalent stress parameter greater than the given stress parameter ($\Delta\sigma_e > \Delta\sigma$). Not surprisingly, the directivity effect is largest for high values of rupture velocity and at distances close to faults with predominantly updip rupture propagation. The latter case is particularly sensitive to directivity because the rays leaving in directions opposite to the rupture propagation are not included in the average.

We have tacitly assumed that the equivalent stress, $\Delta\sigma_e$, is the simulation analogue of the stress parameter found by matching observed ground motions, obtained from many recordings, with theoretical calculations of the ground motions using the point-source, no-directivity stochastic model. The equivalent stress was determined, however, from consideration of the effect of directivity on the high-frequency spectral level, and not on the ground motion parameters themselves. Put another way, would the ground motions predicted by the standard stochastic model, with $\Delta\sigma_e$ substituted for $\Delta\sigma$ in the equations giving the source-spectra (e. g., equation (a) at high frequencies), be equal to those found from averages of the ground motions computed from a Monte Carlo simulation in which the directivity is explicitly accounted for? We expect that most of the effect of directivity is captured by the change in high-frequency level, but to make sure, we performed the simulation experiment just described. We chose a $\mathbf{M} = 6.5$ event, horizontal rupture propagation, and take-off angle between 60° and 120°. Ground motions were computed for 1000 rays at 30 km, using the random process theory described in Boore (1983, 1986) and Joyner's (1984) equations for the source spectrum, modified for directivity by multiplying his lower-frequency corner (determined using the specified value of $\Delta\sigma$) by the directivity factor D. The differences between the average of the logarithms of the ground motions and the logarithms of the motions computed using the equivalent stress (taken from Table 1 , with $\Delta\sigma = 50$ bars and $\gamma = 1.5$) are given in Table 2. The differences are largest for response spectra with frequencies in the range spanned by the corner frequencies of the simulate event; for the higher frequencies the differences are inconsequential. Our conclusion is that the assumption stated at the beginning of this paragraph is valid for the ground motions at frequencies of most interest to engineers and that the answer to the question that we posed earlier is effectively *yes*.

The results in this note suggest that stress parameters determined for earthquakes depend on the amount of directivity incorporated into the observations (and not accounted for in the model fitting used to derive the stress parameters). Observations from different types of waves or from different distributions of stations can lead to

TABLE 1

RATIO OF STRESS PARAMETERS ($\Delta\sigma_e/\Delta\sigma$)

Takeoff Angles	Gamma	Ratio of Rupture to Propagation Velocity									
		0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
		Rupture Direction = 0									
60–120	1.0	1.1	1.1	1.2	1.2	1.2	1.3	1.4	1.4	1.5	1.7
	1.5	1.1	1.2	1.2	1.3	1.4	1.5	1.6	1.7	1.9	2.2
120–180	1.0	1.0	1.0	1.1	1.1	1.1	1.1	1.1	1.1	1.2	1.2
	1.5	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.3	1.3
		Rupture Direction = 45°									
60–120	1.0	1.1	1.1	1.1	1.2	1.2	1.2	1.3	1.3	1.4	1.5
	1.5	1.1	1.1	1.2	1.2	1.3	1.3	1.4	1.5	1.6	1.8
120–180	1.0	1.4	1.5	1.6	1.7	1.8	1.9	2.1	2.2	2.5	2.8
	1.5	1.7	1.8	2.0	2.2	2.4	2.6	2.9	3.4	3.8	4.6
		Rupture Direction = 90°									
60–120	1.0	1.0	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.3	1.3
	1.5	1.1	1.1	1.1	1.2	1.2	1.2	1.3	1.3	1.4	1.5
120–180	1.0	1.7	1.8	1.9	2.1	2.3	2.4	2.7	3.1	3.6	4.3
	1.5	2.1	2.3	2.6	3.0	3.3	3.9	4.5	5.5	6.8	9.5

TABLE 2

DIFFERENCE OF LOG GROUND MOTIONS, CALCULATED FROM MONTE CARLO SIMULATION AND FROM EQUIVALENT STRESS PARAMETER

	v/c	v/c
velocity ratio:	0.70	0.95
effective stress:	70	110
ground motion	difference of log motions	
peak acceleration	0.00	0.00
peak velocity	0.00	−0.01
psv-0.10Hz	−0.05	−0.03
psv-0.16 Hz	−0.06	−0.16
psv-0.25 Hz	−0.01	−0.11
psv-0.32 Hz	0.01	−0.08
psv-0.40 Hz	0.01	−0.07
psv-0.63 Hz	0.00	−0.04
psv-1.00 Hz	0.00	−0.02
psv-3.16 Hz	0.00	−0.01
psv-10.00 Hz	0.00	−0.01

apparent variations in the stress parameters, even if the actual stress parameter is constant. The differences can be quite large (well over a factor of 2). The results also indicate that stress parameters determined from averages of many data (as obtained from regression analyses) should not be used in modeling studies of finite ruptures without correction for the directivity effect implicitly included in the average data. Table 1 is a first attempt at establishing the correction factors.

The effects of directivity discussed here were calculated for a line-source model with unidirectional rupture propagation. We chose this model for practical reasons: a more realistic Monte Carlo simulation in which rupture occurs over extended fault planes would be prohibitively expensive. It might be thought that circular or bidirectional rupture propagation would act to decrease the directivity effect. While the maximum effect as a function of azimuth and take-off angle might be greater for a unidirectional rupture than for a circular or bidirectional rupture, the effect averaged over forward and backward azimuths would undoubtedly be less. (The destructive interference at backward azimuths that is a prominent feature of the unidirectional model (Boore and Joyner, 1978) would not be present in the circular or bidirectional ruptures.) For this reason, the effect of directivity on the averages used to construct Table 1 would be increased for the more realistic fault.

A major assumption in our analysis is that directivity does indeed affect high-frequency spectral levels in a manner similar to that expressed in equation (2). Because of a variety of masking mechanisms and a general lack of data covering the whole focal sphere, there is little unambiguous observational evidence for the effects of directivity at high frequencies. A review of such evidence is beyond the scope of this paper. We see no reasonable way to escape strong directivity effects, however, if (a) the zone immediately behind the rupture front is the source of the largest ground motion amplitude at high frequencies, (b) the rupture front progresses in a more or less orderly way from the hypocenter to the far edges of the rupture, and (c) the average rupture velocity is a substantial fraction of the shear-wave velocity. At our current level of knowledge, all of these assumptions seem reasonable.

Acknowledgments

Critical reviews by Jack Boatwright, Heidi Houston, Charles Mueller, and John Vidale are highly appreciated. This work was supported by the Nuclear Regulatory Commission.

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Manuscript received 10 April 1989