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ON THE m_N, M RELATION FOR EASTERN NORTH AMERICAN EARTHQUAKES

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ABSTRACT

A stochastic model of ground motion has been used as a basis for comparison of data and theoretically-predicted relations between m_N (commonly denoted by m_{bLg}) and moment magnitude for eastern North America (ENA) earthquakes. m_N magnitudes are recomputed for several historical ENA earthquakes, to ensure consistency of definition and provide a meaningful data set. We show that by itself the magnitude relation cannot be used as a discriminant between two specific spectral scaling relations, one with constant stress and the other with stress increasing with seismic moment, that have been proposed for ENA earthquakes.

INTRODUCTION

We recently published a paper (Boore and Atkinson, 1987, hereafter referred to as "BA87") describing the prediction of ground motions and response spectra at hard-rock sites in eastern North America (ENA) . The basis for the predictions was the stochastic model initially proposed by Hanks and McGuire (1981), in which the ground motion is represented as filtered white Gaussian noise of specified duration; the filter function is derived from seismological models of the earthquake source and the effects of propagation to the site. For purposes of prediction, the basic input parameter describing the source size is seismic moment (M_0), or equivalently, moment magnitude (**M**). In ENA, however, a short-period magnitude such as m_N is commonly used as the basic magnitude measure in performing seismic hazard analyses, and therefore a conversion from m_N to **M** is required. We discussed such a conversion in BA87, showing that our theoretical model was in agreement with observations. Despite the agreement, we mentioned that the agreement between theory and data could not be used as a discriminant between spectral scaling relations characterized by a constant stress parameter or by the variable stress parameter implicitly proposed by Nuttli (1983). In the present paper, we elaborate on this assertion, showing the effect of the attenuation model and distance on the predicted relation between the short-period and moment magnitudes. We also take this opportunity to provide more detail regarding our recomputations of the magnitudes.

RECOMPUTATIONS OF MAGNITUDES

In BA87, to compare our theoretical predictions of the relationship between moment and Lg-wave magnitude with available data it was necessary to recompute magnitudes for several of the historic ENA earthquakes. In this section, those recomputed magnitudes quoted in BA87 are documented (and modified somewhat in view of some new information about instrument calibration).

Our theoretical predictions of Lg magnitude use the definition of magnitude originally defined by Nuttli (1973):

$$\begin{aligned} m_N &= 3.75 + 0.90\log D + \log(A/T) \quad 0.5^\circ < D < 4^\circ \\ m_N &= 3.30 + 1.66\log D + \log(A/T) \quad 4^\circ < D < 30^\circ \end{aligned} \tag{1}$$

where A is peak vertical ground displacement in micrometers, T is the period of the peak motion in seconds, and D is the epicentral distance in degrees. (In this paper we adopt the notation m_N for Nuttli magnitude. Nuttli used the notation m_{bLg} because he intended the scale to be equivalent to body-wave magnitude m_b. In BA87 we used the notation m_{Lg}, because we do not believe equivalence to m_b has been established. We have changed to the m_N notation here, however, because it has been suggested [J. Dwyer, pers. comm. 1987] that we may cause confusion with the $m_{Lg}(f)$ measure suggested by Herrmann and Kijko (1983).)

The Lg magnitude measure suggested by Herrmann and Kijko (1983) is:

$$m_{Lg}(f) = 3.81 + 0.833\log D + 48.2GD + \log A \tag{2}$$

where $G = \pi f/\beta Q$, and β is the shear wave velocity and Q the frequency-dependent quality factor. The Herrmann and Kijko formulation was adopted by Toro and McGuire (1987) in their stochastic model predictions, and is appropriate for magnitudes determined on a narrow-band instrument such as the WWSSN seismograph. However, for broad-band instruments such as the early seismographs upon which most of the larger ENA earthquakes have been recorded, or the eastern Canadian digital network (ECTN) seismograph, the Nuttli formulation is preferable. This is because the explicit inclusion of frequency and the assumption of narrow-band response in the Herrmann and Kijko definition renders it inapplicable as a means of obtaining a single average magnitude value to represent an earthquake in such cases. Rather, the Herrmann and Kijko definition will result in a variety of single-station measures, of $m_{Lg}(1)$, $m_{Lg}(0.3)$, $m_{Lg}(0.2)$, etc., depending on the distance and type of instrument. This is not the use which Herrmann and Kijko intended for their scale, and is not particularly helpful in providing a simple size characterization of an earthquake. For these reasons, we used the Nuttli scale as the basis for redetermining historical earthquake magnitudes.

Much previous work on determining m_N and **M** for moderate to large ENA earthquakes has been done by Street and Turcotte (1977). In their determinations of Lg magnitude, however, they chose to apply the restriction that m_N would be computed only if the period of the peak motion was near 1 second. This restriction was suggested by Nuttli as a means of rendering the m_N scale a measure of 1 second energy; an unfortunate consequence was that most of the historical seismograms were then unusable because the peak periods were generally too long.

We have returned to Nuttli's original m_N definition, with no restrictions on period, and recomputed the m_N magnitudes of the historical ENA earthquakes using the station data provided by Street and Turcotte (1977), but recomputing the epicentral distances. Where horizontal-component data are provided, we have assumed $H/V = 1.4$, consistent with the assumption used in our theoretical predictions. The factor of 1.4 was based on a variety of data sources from both eastern and western North America (see Boore and Atkinson, 1987, p. 452). A recent study of ENA 3-component data (Gupta and McLaughlin, 1987) indicates an average H/V ratio of approximately 1.6, with large variations according to site conditions. Hard rock sites exhibit the smallest H/V ratios, and the soft soil sites of the central U. S. exhibit the largest H/V ratios. The calibration constants and magnification curve for each instrument were used to obtain ground displacement from measured amplitude, and this was divided by the period of the peak motion as prescribed by the Nuttli formula. The calculations and results are summarized in Table 1. The entries in the table are largely taken from Street and Turcotte (1977), with a few changes in instrument constants resulting from conversations with R. Herrmann (see his paper in this issue). The calibration of the older instruments remains a major source of uncertainty in the calculation of magnitudes.

To test how well the Nuttli definition succeeds in providing a magnitude estimate that is free from apparent biases, we have plotted the residuals of the individual station estimates against distance, period and magnitude in Figure 1. In this figure, the residual is defined as the difference between the m_N estimate from an individual station and the average m_N for that event from all stations. Perhaps surprisingly, in view of the wide range in distance, magnitude and instrument type, the Nuttli scale does an excellent job; there are no obvious trends in the residuals. We conclude that the Nuttli scale with no restriction on period provides a good measure of earthquake size for moderate to large events recorded on broad-band instruments.

Our revised magnitudes (Table 1) do not differ greatly from those of Street and Turcotte for most of the events; our m_N values generally exceed their m_N(1 sec) values by

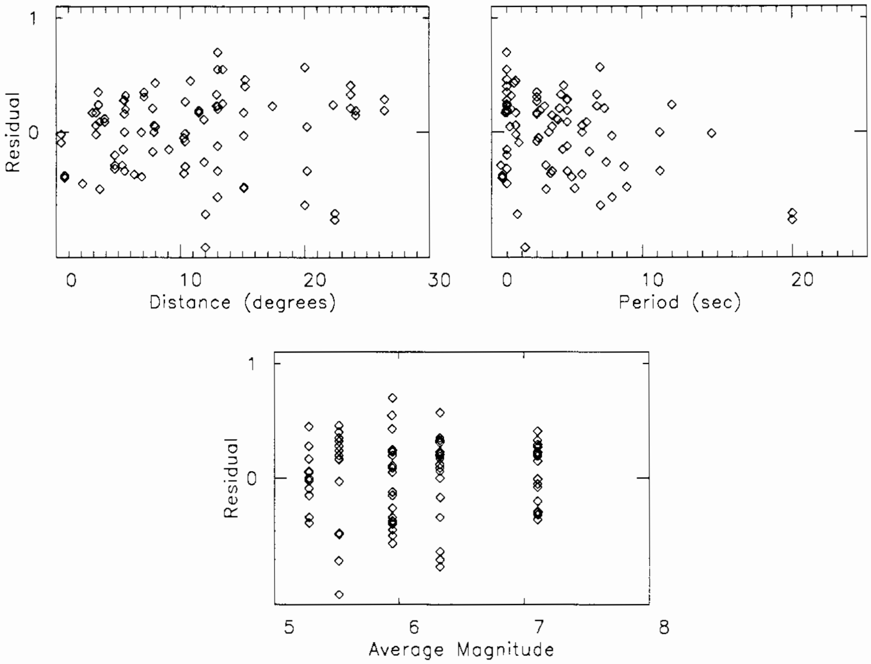

Figure 1. Station residuals for m_N determinations (see text).

0.1 to 0.2 units. For the 1925 Charlevoix event, however, our m_N value is 7.1. which is 0.5 units higher than their 1 second value, and closer to Gutenberg and Richter's m_b estimate from intermediate-period body waves (A. Stevens, personal communication, 1984).

The m_N, **M** data compiled in BA87 also included 8 more recent moderate earthquakes, for which m_N had been determined by the Geological Survey of Canada (GSC). These events were included because the GSC also uses Nuttli's original m_N definition, and thus their definition of magnitude is consistent with what we are using. (GSC does restrict the period of observations to $T < 1.3$ sec; for the moderate events being considered here the predominant periods will be shorter than this.)

Moment magnitudes for the events in our data compilation were drawn from published sources, as described in BA87. We adjusted Street and Turcotte's (1977) moment estimates for consistency with our assumed values of the H/V ratio, average radiation pattern, free surface effect and crustal shear wave velocity, in order to allow unambiguous comparisons with theory. In general, this resulted in increasing their moments by a factor of approximately 1.5.

THEORETICAL PREDICTIONS OF THE m_N,M RELATION

In Figure 2 we compare the m_N, **M** data discussed above to the theoretically-predicted relation between the short-period and moment magnitudes. The theoretical curves are modified from those in BA87 by subtracting 0.1 from the predicted m_N values. This modification is included to account for differences in the definition

Table 1. L_g magnitude calculations for historical earthquakes.

Station	Instr.	Comp.	Dist. (°)	V_0	ς	T_0 (sec)	Amp. (mm)	Per. (sec)	m_N (comp)	m_N (sta)
1925: Charlevoix, Quebec										
CLE	W	EW	10.4	50	0.6	4	22.5	3	7.01	7.19
	W	NS	10.4	50	0.6	4	50	3	7.36	
CLH	BO	EW	10.4	15	0.2	10	26	8.8	6.79	6.93
	BO	NS	10.4	15	0.2	10	29	14.6	7.08	
DEN	W	EW	26.4	50	0.5	5	16	5	7.28	7.33
	W	NS	26.4	50	0.5	5	20	5	7.38	
GEO	W	EW	10.3	115	0.5	8	62.5	3.1	7.04	6.89
	W	NS	10.3	115	0.5	8	40	3.9	6.73	
HAL	W	Z	5.3	50	0.5	5.2	45	3.6	6.82	6.82
NOL	W	EW	23.7	80	0.5	7	54	4.8	7.50	7.41
	W	NS	23.7	80	0.5	7	43	4.6	7.42	
	W	Z	23.7	50	0.5	6	14.5	4.5	7.30	
OTT	MS	NS	4.7	250	0.7	12	50	0.6	6.80	6.82
	BO	EW	4.7	120	0.7	6	37.5	1	6.77	
	BO	NS	4.7	120	0.2	6	51	1	6.89	
SAS	M	EW	24.1	155	0.2	10	45	4	7.24	7.26
	M	NS	24.1	92	0.2	10	22	3.2	7.28	
SLM	W	Z	17.4	50	0.5	5	19.5	3.5	7.32	7.32
								M_N (avg):	7.11	7.11
1929: Attica, New York										
BUF	W	EW	0.4	50	0.5	5	9.1	1.8	5.19	5.22
	W	NS	0.4	50	0.5	5	9.2	1.6	5.25	
CHK	MS	EW	6.9	150	0.7	12	1.7	5.25	4.89	5.08
	MS	NS	6.9	150	0.7	12	3	3.75	5.28	
DEN	W	EW	20.2	50	0.5	5	0.25	4	5.33	5.13
	W	NS	20.2	50	0.5	5	0.1	4	4.93	
HAL	B	?	10.8	120	0.2	5.2	1.5	1.6	5.72	5.72
OTT	MS	EW	3.2	250	0.7	12	10	1.6	5.45	5.35
	MS	NS	3.2	250	0.7	12	7.75	1.6	5.34	
	W	Z	3.2	160	0.5	6	3	1.6	5.25	
SHF	WA	EW	5.4	2800	0.7	0.8	24	1	5.56	5.34
	WA	NS	5.4	2800	0.7	0.8	8.8	1	5.12	
								M_N (avg):	5.28	5.31
1935: Timiskaming, Quebec										
AAM	B	EW	5.5	30	0.2	15	25	11.2	5.99	6.16
	B	NS	5.5	30	0.2	15	55	11.2	6.33	
BOZ	MR	EW	22.4	75	0.6	17	1.75	20	5.62	5.59
	MR	NS	22.4	75	0.6	17	1.5	20	5.55	
BUF	W	EW	3.9	80	0.5	5	85	4.3	6.44	6.43
	W	NS	3.9	80	0.5	5	75	6.3	6.42	
CHI	W	EW	7.8	107	0.2	4.5	34	7.5	6.54	6.35
	W	NS	7.8	96	0.3	4.9	19	6.5	6.16	
CHK	MS	EW	7.9	150	0.7	12	49.2	6	6.39	6.36
	MS	NS	7.9	150	0.7	12	43.2	6	6.33	
CSC	MR	EW	12.9	75	0.6	12	20	7	6.56	6.61
	MR	NS	12.9	75	0.6	12	25	7	6.65	
DEN	W	EW	20.0	50	0.5	5	0.5	7.2	5.69	6.29
	W	NS	20.0	50	0.5	5	8	7.2	6.89	
FLO	WA	EW	11.5	2000	1.0	2	64	1	6.52	6.51
	WA	NS	11.5	2000	1.0	2	61	1	6.50	
WES	W	EW	7.1	50	0.5	5	23	3	6.67	6.66
	W	NS	7.1	50	0.5	5	21.3	3	6.64	
								M_N (avg):	6.33	6.33

(Continued)

Table 1. L_g magnitude calculations for historical earthquakes. (Continued)

Station	Instr.	Comp.	Dist. (°)	V_0	ς	T_0 (sec)	Amp. (mm)	Per. (sec)	m_N (comp)	m_N (sta)
1940: Ossipee, New Hampshire										
BUF	W	EW	5.5	50	0.5	5	7	5	5.79	5.74
	W	NS	5.5	50	0.5	5	3.5	3	5.68	
CGM	WA	EW	15.2	1500	1.0	1.7	8	1	5.97	5.94
	WA	NS	15.2	1500	1.0	1.7	7	1	5.91	
CHK	MS	EW	12.0	250	0.7	12	0.2	2.2	4.51	4.65
	MS	NS	12.0	250	0.7	12	0.3	1.7	4.79	
FLO	WA	EW	15.1	2000	1.0	2	7	3	5.69	5.31
	GW	EW	15.1	800	1.0	12	7.5	9	5.04	
	GW	NS	15.1	800	1.0	12	4.5	5.5	5.02	
	GW	Z	15.1	800	1.0	12	14	8	5.49	
HAL	B	EW	5.6	120	0.2	5.2	4.5	1.3	5.83	5.77
	B	NS	5.6	120	0.2	5.2	3.4	1.3	5.71	
OTT	MS	EW	3.4	250	0.7	12	15	1	5.86	5.81
	MS	NS	3.4	250	0.7	12	11.7	1	5.75	
SHF	WA	NS	2.9	2800	0.8	0.8	65	0.9	5.68	5.68
								M_N (avg):	5.52	5.56
1944: Cornwall, Ontario										
BUF	W	EW	3.5	50	0.5	6	29	5	6.07	5.78
	W	NS	3.5	50	0.5	6	5.2	3.6	5.48	
CGM	WA	EW	13.4	1500	1.0	1.7	36	1	6.53	6.38
	WA	NS	13.4	1500	1.0	1.7	18	1	6.23	
CSC	MR	EW	11.9	75	0.6	12	4.8	4.4	6.09	5.90
	MR	NS	11.9	75	0.6	12	3.6	7.6	5.72	
FLO	GW	EW	13.0	800	1.0	12	21	8	5.41	5.86
	GW	NS	13.0	800	1.0	12	21	5	5.64	
	GW	Z	13.0	800	1.0	12	25	5	5.86	
	WA	EW	13.0	2000	1.0	2	26	1	6.21	
	WA	NS	13.0	2000	1.0	2	24	1	6.18	
GEO	GW	Z	6.3	800	1.0	12	60	6	5.61	5.61
HAL	B	EW	8.0	120	0.2	5.2	11	1.5	6.41	6.22
	B	NS	8.0	120	0.2	5.2	3.6	1.2	6.03	
MLF	GW	Z	9.1	800	1.0	12	39	4.7	5.83	5.83
OTT	MS	EW	0.7	250	0.7	12	22	0.7	5.57	5.56
	MS	NS	0.7	250	0.7	12	21	0.7	5.55	
SAS	MS	EW	22.3	150	0.7	12	8.7	12	6.22	6.22
SHF	WA	NS	2.1	2800	0.8	0.8	57	1	5.53	5.53
SLM	WA	EW	13.0	1000	1.0	1.7	25	1	6.53	6.60
	WA	NS	13.0	1000	1.0	1.7	35	1	6.68	
								M_N (avg):	5.95	5.93

Legend:

Instrument types: B (Bosch), BO, (Bosch–Omari), GW (Galitzin–Wilip), M (Mainka), MR (McComb–Romberg), MS (Milne–Shaw), WA (Wood–Anderson), W (Wiechert).

V_0, ς, T_0 are static magnification, fraction of critical damping, and natural period of instruments, respectively.

All instruments are simple mechanical oscillators except GW, for which the amplification relative to ground displacement is given by $4V_0U/(U^2+1)^2$, where $U = T/T_0$ and T = the observed period of motion. (R. Herrmann, personal communication, 1987.)

Amp. aud Per. are observed maximum sustained zero-to-peak amplitude and period, respectively.

m_N (comp) and m_N (sta) are computed L_g magnitude from each component and averaged over components at a given station, respectively.

m_N (avg) is the L_g magnitude for each event obtained by averaging the entries in the respective column.

of the peak motion: Street and Turcotte (1977) use maximum sustained amplitude (apparently defined as the third highest peak), whereas the theoretical magnitudes are calculated from estimates of the largest peak motion. The correction of 0.1 is based on observational experience (R. Herrmann, oral communication, 1987) and has been confirmed by numerical simulations. The theoretical curves show the implications of two attenuation models published for Lg-wave attenuation in central and eastern North America:

$$Q = 1300f^{0.38}, \tag{3}$$

(Dwyer et al, 1983) and

$$Q = 500f^{0.65}. \tag{4}$$

(Shin and Herrmann, 1987). Figure 2 also shows the effect of distance on the predicted relations, by displaying the predictions at two distances, 400 km and 800 km. (For the larger earthquakes, most of the data used to determine m_N come from distances nearer 800 km than 400 km.) The main thing to notice is that the two theoretical curves in each plot are similar in shape, and that by changing the attenuation other scaling can fit the data. This means that without knowledge of the proper attenuation function, the degree of fit of the theoretical curves to the observations is not a good discriminant between the spectral scaling models. As it stands, the standard model used in BA87, which uses the attenuation model given by equation 4, a distance of 800 km, and a constant stress parameter of 100 bars, is a reasonable fit to the data. The following equation is a good approximation to the curve:

$$\mathbf{M} = 2.689 - 0.252m_N + 0.127m_N^2 \tag{5}$$

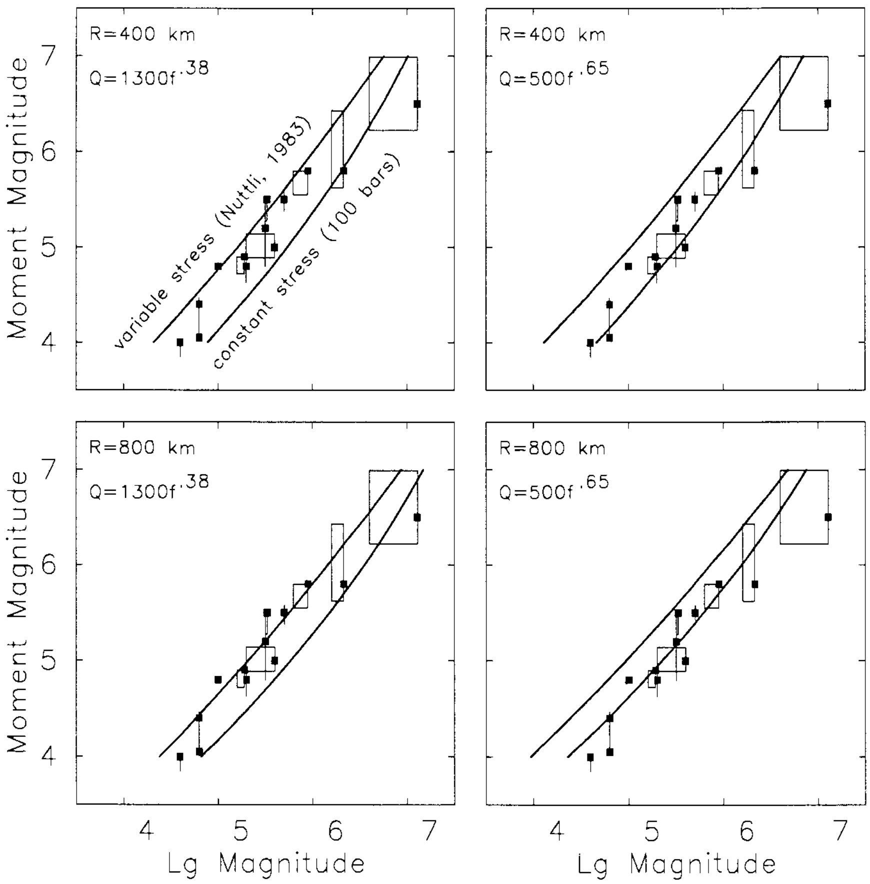

Figure 2. Theoretical predictions (curves) and observations (symbols) of M and m_N. Theoretical predictions, discussed in detail in Boore and Atkinson (1987), are computed as the response of a Wood-Anderson seismometer at distances of 400 km and 800 km for variable and constant stress parameter models. For the data, the solid squares are our preferred values, and the boxes (and line segments) enclose other published estimates.

(this is obtained from equation 12 in BA87 by accounting for the 0.1 correction in m_N discussed above). A straight-line fit to the data,

$$\mathbf{M} = 1.12 m_N - 1.00, \tag{6}$$

was reported by Somerville (pers. comm., 1987); this fit also agrees with the theoretical predictions.

Although the theoretical relation between M and m_N is a relatively sensitive function of attenuation, this is not so for predictions of ground motion amplitudes at distances within a few tens of kilometers of the faulting, and therefore such data provide a much better discriminant between spectral scaling models. BA87 showed that response spectra computed from the small number of accelerograms obtained close to faults in eastern North America was in agreement with predictions using the constant-stress scaling (with a stress parameter of 100 bars) but not with the variable-stress scaling.

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